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The Violation of Bell's Inequality in a Deterministic but Nonlocal Model

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THE VIOLATION OF BELL'S INEQUALITY

IN A DETERMINISTIC BUT NONLOCAL MODEL

by

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GRADUATE COMMITTEE APPROVAL

of a thesis submitted by

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As chair of the candidate's graduate committee, I have read the thesis of Stephanie A. Magleby in its final form and have found that (1) its format, citations, and bibliographical style are consistent and acceptable and fulfill university and department style requirements; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory to the graduate committee and is ready for submission to the university library.

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ABSTRACT

THE VIOLATION OF BELL'S INEQUALITY

IN A DETERMINISTIC BUT NONLOCAL MODEL

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Master of Science

This thesis investigates the violation of Bell's Inequality through the use of nonlocal measurement schemes as encapsulated in a quasi-deterministic toy model. This toy model, called the Q Box, is reminiscent of Mermin's Box in that it describes a system that appears to be deterministic yet produces the statistics of a quantum type system.[1] [1] The workings of the Q Box are detailed both as a thought experiment and as a computer simulation. Nonlocal measurement protocols similar to those which generate a violation of Bell's Inequality in the Q Box are also applied to Mermin's Box, with comparable results.

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[1] N.D. Mermin, "Bringing home the atomic world: Quantum mysteries for anybody," Am. J. Phys. 49: 940-943 (1981). N. David Mermin, Am. J. Phys. 58:731-734 (1990).

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Thank you, Dr. Tolley. You were the bridge to the land of probabilities and the driving force that kept this all going. You were the one who first posed the questions. Thanks for including me in the quest for answers.

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Most of all, thank you to my family for being willing to sacrifice so that I can fulfill this dream. Thank you, Spencer, for listening and encouraging and loving me through it all. Thank you, Sterling, Hans, Clarissa and Austin, for being my best cheerleaders during this long process. I could not have done it without you.

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Background

The term "entanglement" was first coined by Erwin Schrödinger in 1935. Entanglement is, as he put it, "not *one* but rather *the* characteristic trait of quantum mechanics [1]." We wish to investigate entanglement and nonlocality, which is closely related, by encapsulating them in a model that is deterministic yet produces the statistical signature of a quantum system.

1.1 Entanglement

Entanglement is like knowing two twins who are so completely identical in looks, disposition, mannerisms, opinions, etc. that one can never be sure which is which. But if you meet them at a party and find that the twin by the punch bowl is Bob, then the one next to the fireplace is definitely Ted. This example is an oversimplification of entanglement which will be explored more fully later in this chapter.

A classic example of an entangled system is that of a photon that has been down-converted. The original experimenters in the field were Leonard Mandel and R. Ghosh of the University of Rochester. In the down-conversion process they pioneered, a 'mother' photon passes through a nonlinear crystal and is divided into two 'daughter' photons which each have one-half the energy of their progenitor. The daughter photons go off in two different predictable directions, but each carries with it a trait that is linked inexorably to its twin. For example, if one of the pair is right circular polarized upon exiting the crystal, then the other is guaranteed to be found upon measurement to be left circular polarized, and vice versa. [2] Hence we refer to the polarization as being correlated between the twin pair. Other measurable characteristics show such correlations. For example, one can have particles which are entangled spin-up/spin-down, right/left polarized, in their angular momentums, etc. In all of these cases of entanglement it is the correlation between the two particles that is makes it unique.

One of the most popular examples of entangled systems in the physics literature is that of spin-½ particles and was first conceived by David Bohm. [3] Written as a superposition of amplitudes, this correlated state is that of a singlet:

$$\psi = \frac{1}{\sqrt{2}}\left[(\uparrow)_1(\downarrow)_2 - (\downarrow)_1(\uparrow)_2\right] \tag{1.1}$$

Mathematically, Eq. (1.1) describes a system of two entangled particles in which either the first one is spin-up and the second one is spin-down ($(\uparrow)_1(\downarrow)_2$) or the first one is spin-down and the second spin-up ($(\downarrow)_1(\uparrow)_2$). In terms of our earlier twin analogy, one could say that either the twin at the punchbowl is Bob and therefore the man at the fireplace is Ted, which would be expressed mathematically as $(B)_1(T)_2$, or the twin at the punchbowl is Ted and the other one Bob, written $(T)_1(B)_2$; but we can't be sure because they are so much alike, making the likelihood of finding Bob at the first location (the punchbowl) and Ted at the second location (the fireplace) in the absence of any further information about their respective proclivities:

$$\left|\Psi^{*}\Psi\right| = \frac{1}{2}\left[(B)_1(T)_2 - (T)_1(B)_2\right]^2 \tag{1.2}$$

Of course, this is a very simplistic example and assumes we know nothing about each man's drinking habits or personal punch preferences or whether or not they enjoy standing by fireplaces.

1.2 The Copenhagen Interpretation

There is an important difference between the standard quantum interpretation (also known as the Copenhagen[2] or Orthodox[3] Interpretation) of two entangled particles in quantum mechanics and the previous illustration of two indistinguishable twins at a party. In the Copenhagen Interpretation, the two entangled particles are considered to be in a superposition of the state of spin-up *and* the state of spin-down until measurement, as described mathematically in Eq. (1.1). The *meaning* of Eq. (1.1) is a matter of personal interpretation. Is Eq. (1.1) a complete description of the state of nature or a mathematical representation of the probability of a particular outcome given a measurement?

Before measurement, a physicist with a Copenhagen bent assumes that the particle has not yet been 'realized.' In other words, until forced by measurement to make a decision, a Copenhagener particle exists in a fuzzy probabilistic state of potentialities; a nebulous state of either/or. This is a different case than the two twins at the party. Bob

[2] So called because the main champion of this interpretation was Niels Bohr, a Danish physicist who lived and worked in Copenhagen. Other scientists, including Werner Heisenberg and Max Born, also contributed.

[3] The religious allusion is apropos. Quantum mechanics works mathematically and experimentally, but the philosophical *implications* assigned to the experiments are more a matter of taste and inclination.

knows (assuming he hasn't had too much to drink) that he is standing by the punchbowl and that his name is and always has been Bob. Ted also had, has and ever will have an identity separate from his twin. The confusion about the identity of Bob and Ted arises from the lack of discriminating characteristics, not any inherent question of 'Bobness' or 'Tedness'.

By comparison, under the Copenhagen interpretation, the two entangled particles do not know whether they are spin-up or spin-down until a measurement is made on either of the particles. According to the strictest Canon of Quantum Orthodoxy, before measurement the particles can not even be said to be in a state of 'either/or,' they are considered to be in a state of 'both.' To add to the weirdness, *immediately* upon the measurement (and subsequent realization) of one particle, the spin state of the second particle also manifests its spin; like two fuzzy pictures pulled instantly into focus at the same moment in time, wherever they are in space. In the physics literature this measurement-induced state is called "the collapse of the wave function," and it happens *even if the two are separated light-years across the galaxy*. This is nonlocality, or what Einstein called "spooky action at a distance." [4]

1.3 The Debate

These features of entanglement and nonlocality, combined with the concepts of uncertainty, wave/particle duality and the central role of the observer, combine to make the philosophical implications of the Copenhagen Interpretation unsettling.[4] Einstein

[4] Provocative, tantalizing, astounding, unearthly, bizarre, uncanny, strange—these are all words used by prominent physicists to describe the counter-intuitive results of quantum mechanics. It is hard to avoid flamboyant words when even wikipedia uses verbage such as: This effect is now known . . . colloquially as "quantum weirdness". [5]

found it to be particularly hard to swallow and set about thinking up scenarios and Gedanken experiments to refute the Copenhagen model. His most well-known argument was embodied in the 1935 paper entitled "Can a Quantum-Mechanical Description of Physical Reality Be Considered Complete." [6] Over time this paper has become known as "the EPR Paper" for its three authors: Albert Einstein, Boris Podolsky and Nathan Rosen. The first entangled system was introduced in the EPR paper, although the authors never used that term.

Einstein, Podolsky and Rosen argued in their EPR paper that every complete theory should have three components:

1) It should be deterministic, even if one must resort to using probability due to too little or too much information.
2) It should describe "elements of reality." Attributes of a system can be measured with certainty.
3) It should be local. Nonlocality was unpalatable to Einstein.

If these three attributes were not all present, Einstein considered the theory to be "incomplete"—and quantum mechanics definitely fit into that category in Einstein's opinion. He had this to say about the reservations the Copenhagen Interpretation fostered in him [7]:

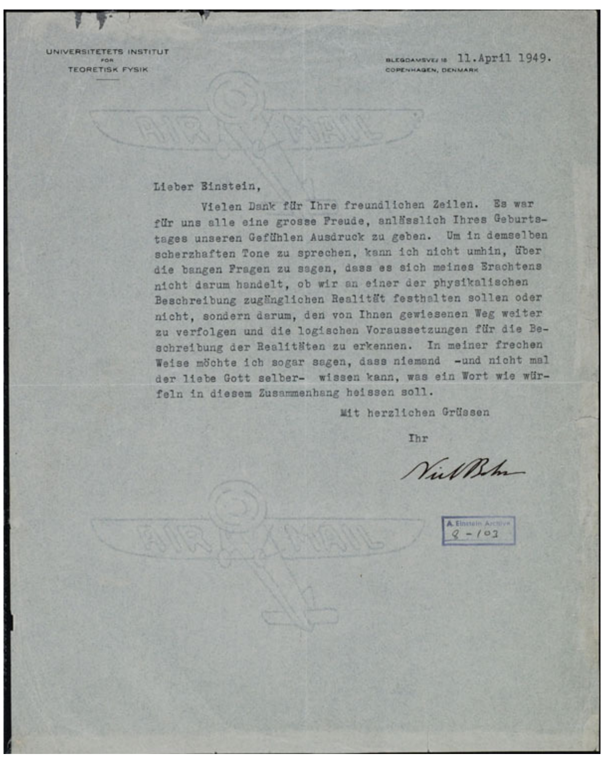
> *One wants to be able to take a realistic view of the world, to talk about the world as if it is really there, even when it is not being observed. I certainly believe in a world that was here before me, and will be here after me, and I believe that you are part of it! And I believe that most physicists take this point of view when they are being pushed into a corner by philosophers.*

The source of one of the most famous quotations attributed to Einstein was a letter to Max Born in 1926 where Einstein said:

Quantum mechanics is certainly imposing. But an inner voice tells me it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the Old One. I, at any rate, am convinced that He does not throw dice.[8]

Born replied, “Stop telling God what He must do!” [8]

The debate between Einstein and Bohr lasted their entire lifetimes. While taking the opportunity to thank Bohr for sending him 70th birthday wishes, Einstein took the opportunity to gently needle his friend and opponent. He wrote, "This is one of the occasions, which is not dependent on the disquieting question if God throws the dice or if we should hold on to the available physical description of realities...." Here is a photo of Bohr’s reply, followed by its translation [9]:



UNIVERSITETETS INSTITUT FOR TEORETISK FYSIK

BLEGDAMSVEJ 15 COPENHAGEN, DENMARK 11.April 1949.

Lieber Einstein,

Vielen Dank für Ihre freundlichen Zeilen. Es war für uns alle eine grosse Freude, anlässlich Ihres Geburtstages unseren Gefühlen Ausdruck zu geben. Um in demselben scherzhaften Tone zu sprechen, kann ich nicht umhin, über die bangen Fragen zu sagen, dass es sich meines Erachtens nicht darum handelt, ob wir an einer der physikalischen Beschreibung zugänglichen Realität festhalten sollen oder nicht, sondern darum, den von Ihnen gewiesenen Weg weiter zu verfolgen und die logischen Voraussetzungen für die Beschreibung der Realitäten zu erkennen. In meiner frechen Weise möchte ich sogar sagen, dass niemand -und nicht mal der liebe Gott selber- wissen kann, was ein Wort wie würfeln in diesem Zusammenhang heissen soll.

Mit herzlichen Grüssen

Ihr

Niels Bohr

A. Einstein Archive 8-103

Dear Einstein,

Many thanks for your kind letter. For all of us it was a great pleasure to express our sentiments on the occasion of your birthday. To continue in the same jocular tone, I have no choice but to say on this painful issue that it really doesn't matter, in my opinion, whether or not we should hold on to the physical description of accessible realities or further pursue the path you have shown and recognize the logical assumptions for the description of realities. In my impertinent way I would suggest that nobody—not even God himself—would know what a phrase like playing dice would mean in that context.

With friendly regards
Your
Niels Bohr

Figure 1-1 Letter from Bohr to Einstein

1.4 Bell's Theorem: An Alternative to Arguing

In the 1960's, John Bell published two papers: "On the Einstein-Podolsky-Rosen paradox," [10] and "On the problem of hidden variables in quantum mechanics." [11] If not for John Bell and the excellent theorem he presented in these papers, the debate might still be raging. Bell realized that Einstein had defined succinctly the essence of the quantum paradox; one must either assume that quantum theory is incomplete, or be willing to give up either realism or locality. [12] [13] [14] Instead of assuming that quantum mechanics is incomplete, Bell chose instead to test for locality. He formulated an inequality based on three binary measurements made on a system that has local, hidden variables and combinations of these measurements are compared. These hidden variables are the kind that Einstein would have approved of; embodying local reality.[5]

The violation of Bell's Inequality, therefore, is proof of nonlocal behavior such as that demonstrated in some cases by quantum systems, specific examples of which will be given in Section 3.4 of this thesis. It could be argued that violation of Bell's Inequality could conceivably follow from an assumption other than locality upon which Bell's Theorem is based, i.e. realism or determinism. Henry P. Stapp [8] describes the implications of Bell's Theorem to quantum mechanics and then systematically identifies each hypothesis implicit in the derivation of Bell's Theorem. He then shows that for every assumption except nonlocality, the Inequality can still be violated.

[5] There are those who disagree with this characterization of Einstein's thought processes as represented in the EPR paper and other writings. Max Jammer, in his book *The Philosophy of Quantum Mechanics*, states "One of the sources of erroneously listing Einstein among the proponents of hidden variables was probably J.S. Bell's widely read paper: On the Einstein-Podolsky-Rosen Paradox. . . Einstein's remarks . . . are certainly no confession of the belief in the necessity of hidden variables. " We disagree with Jammer. A more complete discussion is available. [15], [16], [17]

Bell's Theorem takes the question of whether quantum mechanics is incomplete or simply statistically extraordinary out of the debate forums and into the lab, where the question can be made an experimental rather than a philosophical one. Here is Bell's Inequality:

$$\left|E_{HV}\left(\hat{a},\hat{b}\right)-E_{HV}\left(\hat{a},\hat{c}\right)\right|\leq 1+E_{HV}\left(\hat{b},\hat{c}\right) \tag{1.3}$$

In Bell's inequality, $E_{HV}\left(\hat{a},\hat{b}\right)$, $E_{HV}\left(\hat{a},\hat{c}\right)$ and $E_{HV}\left(\hat{b},\hat{c}\right)$ represent the results of three combinations of measurements done on particles emitted as pairs in different directions using, for example, spontaneous parametric down-conversion. These particles are assumed to have local hidden variables. Each particle is caught and measured in an analyzer. $E_{HV}\left(\hat{a},\hat{c}\right)$ is the expectation value of a joint measurement on the pair of particles. This measurement is made with one analyzer oriented in the $\hat{a}$ direction and the other oriented in the $\hat{c}$ direction, and so on. For some orientations of $\hat{a}$, $\hat{b}$ and $\hat{c}$ the measurements expected from a system with local hidden variables match those from a quantum system. For other orientations, though, the inequality is violated, leading Bell to conclude that there are no possible *local* hidden variables that can exist to explain the contradiction. Something else is at work. The "something else" is nonlocality, Einstein's bugaboo.

1.5 Overview

The focus of this research is to construct an entangled yet classical system with nonlocal attributes.[6] Our system, which we have named the Q Box, is modeled after Mermin's Box. Mermin's Box is a pedagogical example designed by N. David Mermin to illustrate the more unintuitive aspects of quantum systems. Mermin actually published at least three different versions of his box [10], but we will focus on his first one in Chapter 2. Chapter 3 of this thesis will focus on Bell's Theorem and the role it plays in identifying quantum systems. This chapter will also include a description of hidden variables. We will take a short side trip into the world of cellular automata and examine the measurement protocols in the Game of Life in Chapter 4. Following this, a description of the design of the Q Box will be given in Chapter 5, along with a discussion of nonlocality and how it is embedded into the Q Box system. In Chapter 6 we will return to our analysis of Mermin's Box, using the techniques developed for our Q Box. Chapter 7 is our 'reality check'. The presentation of a computer model of the Q Box, along with a look at how marginalization changes the probability outcomes in quantum structures will be contained in Chapter 8. A comparison of our nonlocal, hidden variable theory with Bohmian Mechanics and the Copenhagen Interpretation will be contained in Chapter 9. Conclusions come finally in Chapter 10.

[6] We realize that the term 'classical nonlocal system' could be considered an oxymoron and beg patience from the reader.

Chapter 2

Mermin's Black Box

In 1981 N. David Mermin [13] described in the *American Journal of Physics* an imaginary black box device. Mermin intended his Box to be a sort of mental logic game that, when played, would serve to introduce the physics layman to what Mermin called "the metaphysical implications of the Einstein-Podolsky-Rosen conundrum," as manifested in the uncanny, seemingly inexplicable correlations discussed in Chap. 1 of this thesis. The object of the game is to mentally manipulate his box, observe its behavior, and deduce the contents of the box and the mechanism by which it works. In the beginning, Mermin tells his reader that his device models "the known behavior of matter on the atomic level," although the title of his article, "Bringing home the atomic world: Quantum mysteries for anybody" should be sufficient warning even to the layman to expect the unexpected.

Mermin's Gedanken device is ingeniously designed. The "matter on the atomic level" it specifically models is the behavior of spin-½ particles in the singlet state. Bohm and Aharonov [3] were the first to use spin-½ particles in an alternative rendition of the Einstein-Podolsky-Rosen Gedanken experiment. In their version of EPR, they use spin-up and spin-down particles as produced in a Stern-Gerlach experiment instead of the momentum and position variables of the original. The Bohm version of EPR has become so popular in the literature that his name often gets tacked onto the end, as in "the

Einstein-Podolsky-Rosen-***Bohm*** Gedanken experiment [13]," even though Bohm conceived of his spin version of EPR twenty-two years after Einstein, Podolsky and Rosen published their paper in 1935.

Mermin [13] says of his hypothetical box:

> *Although this device has not been built, there is no reason in principle why it could not be, and probably no insurmountable practical difficulties.*

In principle, this is true, as manifested in quantum optics labs all over the world. But Mermin initially describes his box as if it were a mechanical object, full of sensors and relays acting deterministically. It is not until the end of the game that he lets us peek inside the black box to find that the "mechanism" is strictly quantum, and not a deterministic system. Is it possible to combine these two versions of Mermin's Box; the naively mechanical version and the quantum object? In other words, is it possible to build a box displaying quantum statistics with classically deterministic and mechanical inner workings?

We think so. To be successful, we must replicate the entanglement features of spin-½ particles without resorting to quantum processes such as down-conversion. Since we believe that nonlocality plays a fundamental role in quantum entanglement, we must also find a way to add nonlocal effects to our buildable box.

Before we build our box, we must understand the model we are trying to duplicate. Let's start by looking closely at Mermin's imaginary device.

2.1 An overview of Mermin's Gedanken Gadget

Mermin imagined a simple system comprised of three components: two detectors and one source. The detectors are completely isolated from each other, and are in no perceivable way connected. This means that there are no electrical wires running between them, no way to produce or receive sound signals from each other, no means of transmitting or detecting vibrations along the surface upon which they both rest. In short, the detectors' only input is a particle from the source. The only output from the detectors is the flashing of a red or green light in response to the arrival of the particle from the source. An outside subjective observer is necessary to interact with the system by noting the color of the flashing light and the setting of the detectors. No other information can be exchanged in any way

2.2 The General Layout of Mermin's Box

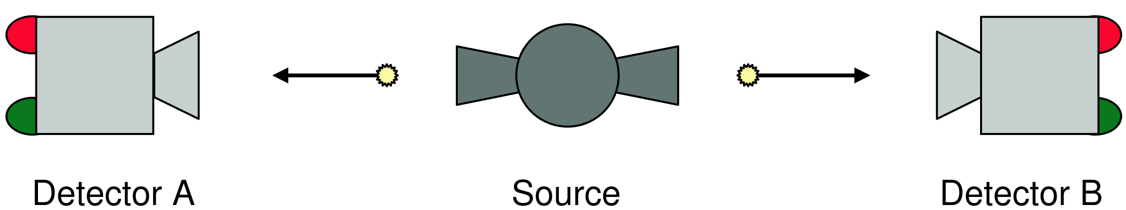


Figure 2-1 The general layout of Mermin's Box

As shown above, the source sits between the two detectors and, when triggered, produces two particles instantaneously, one to each source. This requirement of "instantaneousness" assures that there can be no transmission of information from a newly birthed particle back to the source; no way to send one particle scouting ahead to see what conditions are like so that the second particle can be designed appropriately. These perfect twins burst into existence simultaneously and immediately part ways, each eventually going to its ultimate fate of measurement in its assigned detector.

There are no spatial or geometrical requirements for the positioning of the detectors and the source. They can be placed any arbitrary distance apart and need not be equidistant from each other. In principle, it is possible to leave one detector in the same room as the source while placing the second detector on the other side of the galaxy. This makes it possible for one detector to trigger before the other detector has received its particle.

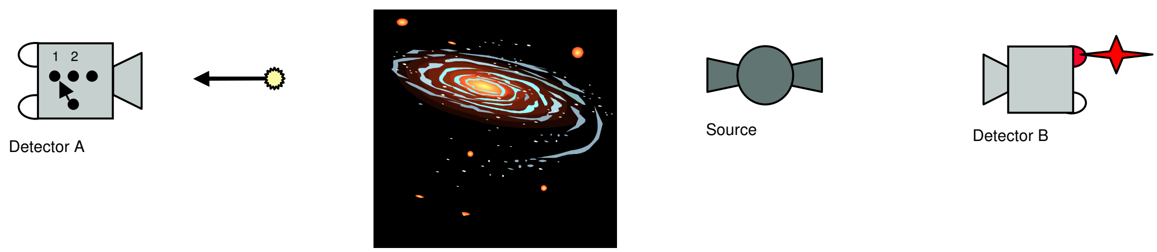


Figure 2-2 In principle, one detector can be any arbitrary distance from the other.

It is also perfectly acceptable to change the respective distances of the detectors to the source between data events, allowing first one detector to be closer to the source and then the other in some random pattern within the same data collection run. We wish to emphasize that although the particles are created at exactly the same moment in time, the distance each travels and therefore the defining moment of measurement for each can be very different.

2.3 The Detectors

Now that we have described Mermin's source box, the production of the particles and the general layout of the Gedankenexperiment, it is time to examine the detectors more closely. Besides the green and red lights that serve to announce the arrival of the particle, there is on the top of each detector a switch that can be turned to one of three possible settings.

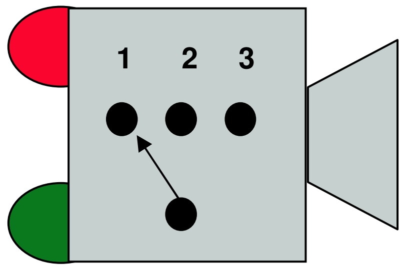


Figure 2-3 Each detector has three possible settings and can signal either red or green.

Just as the detectors can be moved random distances relative to each other and the source, the settings on the detectors can also be changed between individual detection events. The detector settings can even be changed *after* the source has emitted the particles while the particles are in flight. If the respective distances between each detector and the source are different, it is also acceptable to change the setting on one detector *after* the other detector has already fired its response to the source but *before* a signal could be transmitted from the already fired detector to the untriggered one. The

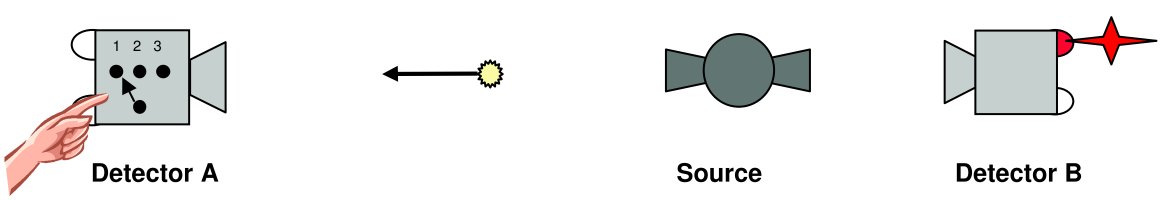


Figure 2-4 The settings of one detector can be changed even after the other detector has fired.

only requirement in this case is that the difference in the distances, detector to source, be such that there is time to mechanically change the switch while awaiting the arrival of the later particle. These constraints should be sufficient to ensure that there is no way to cheat the system; no possible way to exchange information between the two particles.

It is not clear at this point in our examination of Mermin's Box what exactly the detector settings do. It seems reasonable to expect that the settings change the measurement process in some way, but how exactly remains to be seen. In the beginning we have no way of knowing what exactly the source particle is. We also have no information about the mechanism by which the detector decides which color to flash. Our goal is to analyze the patterns of the flashing lights, compare the data to the detector settings and then figure out how the box works.

Since there are three possible settings on each of two detectors, there are nine possible setting combinations between the two detectors: 11, 12, 13, 21, 22, 23, 31, 32, and 33. For example, the combination 13 means that detector A is set to position 1 and detector B is set to position 3, like so:

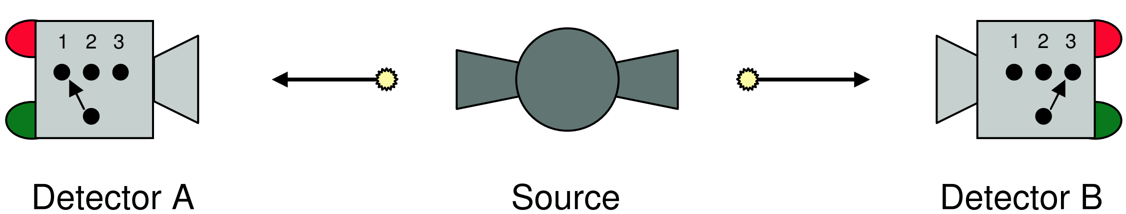


Figure 2-5 One possible setting configuration.

2.4 The Data

Each time the source is triggered, the detector settings and responses are recorded. In Mermin's 1981 *American Journal of Physics* article [13] he provides an illustration of possible data from his device. Remember, Mermin is making this up! His data is a fabrication; but it is constructed such that it matches the data expected from entangled spin-½ particles. Mermin himself tells us so.

23GR	12GR	11GG	13GR	23RG	33GG	12RG	21GR
11RR	31RG	22GG	11GG	22GG	11RR	32RG	23RG
31RG	32GR	22RR	31RG	13RG	22GG	22RR	12GG
33RR	22RR	21GR	32RG	11GG	32GR	33GG	21GR
32GG	22GG	11RR	11GG	23GG	12RR	32GR	11GG
32RG	12RG	13RG	33GG	21RG	13GR	31RR	32GR
31GR	13GR	21RG	33RR	13GR	11RR	11GG	13RG
12GG	32GR	33GG	21GR	21GG	33RR	23RG	21GG
13GR	11GG	32GG	31GR	32RG	33RR	13RR	13RG
11GG	31RG	33RR	12RG	21GR	11GG	22GG	33GG
33RR	21GG	21RG	12RG	11GG	12RG	11GG	23GG
11GG	12RR	12RG	31GR	23GR	12GR	33GG	31GG
11RR	22RR	12RG	22GG	23GR	12GR	23RG	21RR
13GR	21RR	33RR	33RR	13RG	23RG	33GG	32RR
13RR	32RG	11RR	11RR	11RR	32RG	12RG	21RG
23RG	23RR	21RG	33RR	13GR	12GR	23RG	21RR
13RR	21GR	12RR	31GR	12RG	13GR	13RG	22RR
21RG	23GR	11RR	12RR	33RR	21RG	13GR	21RR
31GR	23GG	13RG	21RG	11GG	12GR	23GR	13GG
11RR	31RG	11GG	31GR	32GR	32RG	32GR	11GG

Figure 2-6 A Gedanken data set for a Gedankenexperiment. Reprinted from Am. J. Phys., (49), 942, (October 1981).

The table from Figure 2-6 is taken directly from Mermin's paper. [13] The notation 11GG (bottom right-hand corner) corresponds to the data event where both detectors were turned to position 1 and both responded by flashing green, whereas 23GR (upper left-hand corner) signifies that detector A was set to position 2, detector B was set to position 3, and their respective response flashes were green and red. The setting on detector A is always listed first (**2**3GR) and the setting on detector B second (2**3**GR), even if their flashes did not appear in that order in time. The same is true for the listing of the colored responses; the color immediately following the detector setting numbers (23**G**R) is the feedback from detector A, while the last letter in the series (23G**R**) designates the output from detector B.

Mermin's table is a small, hypothetical sampling (remember, this is a Gedankenexperiment!) of a much larger collection of data. The only way to access the mystery of the box is to observe the output and then examine the statistical behavior of the system. Reliable statistics require sufficient data, and we will assume in our exploration of this imaginary data set that it is not only of sufficient size to render a viable stochastic model, but that it is perfectly collected and recorded, without human error. We wish to stress that although the data set Mermin conjured up is contrived, it statistically matches both the predictions made by quantum theory and the vast amounts of data collected by experimentalists doing photon polarization experiments like those of Aspect and co-workers [19].

The fictional data accumulated from Mermin's imaginary machine appears at first to be statistically random, but close examination reveals two rules of behavior. When the detectors are set to the same numbered settings, i.e. 11, 22 or 33, the detectors *always* flash the same color. However, *which* color, red or green, is left unspecified. During one data event a joint setting of 11 might produce flashes of red-red while the same joint setting of 11 in the next data event might produce flashes of green-green.

The second rule of behavior relates to what happens when the detectors are set differently. When the settings don't match, as in the possible choices 12, 13, 21, 23, 31 and 32, Mermin tells us that the detectors flash the same color about 25% of the time, and flash different colors about 75% of the time. Following is a table summing up the results from Mermin's small sample set.

	RR	GG	Identical Flashes	RG	GR	Different Flashes	Totals
Identical Settings	**25**	**27**	**52/52 Or 100%**	**0**	**0**	**0**	**52**
Different Settings	**14**	**12**	**26/108 Or 24.01%**	**44**	**38**	**82/108 Or 75.99%**	**108**

Figure 2-7 An overview of the behaviors exhibited by Mermin's Box

We will use the two behaviors extracted from the data to try to glean some information about the inner workings of the box. These two behaviors can be condensed thus:

> ***Rule One***: *Every time the detectors are identically set they flash the same color, although* ***which*** *color is impossible to predict.*

***Rule Two**: When the detectors are turned to different settings, they flash the same color about 25% of the time.*

Rule One, matching colors for matching settings, suggests that the particles are created equally, and that they carry within them, encoded from their birth, a set of instructions on how to behave upon meeting a detector. But since there are three possible settings the particles can encounter, the instruction set must be a triplet, and both particles must carry the same three bit information set. This seems to be the only logical explanation for why the detectors flash the same color when set identically.

2.5 The Mechanism

For further help in understanding what might be going on in Mermin's Box, we would like to call on Maxwell's demon. James Clerk Maxwell, father of electromagnetism, originally conjured up his demon as a device to help understand entropy in thermodynamic systems. Maxwell imagined a little impish figure, nimble, intelligent and the size of a gas particle. Maxwell inserted his demon into various thermodynamic scenarios and used him to mentally manipulate the system in ways that an experimenter could not. Maxwell's demon, then, is also a Gedanken gadget. We will use Maxwell's Gedanken gadget to explore Mermin's Gedanken gadget by imagining our demon sitting inside the source box, perhaps holding a clipboard and wearing a whistle like a 7th grade gym teacher. As each new pair prepares to exit he barks out orders:

"All right, you two! You are Red-Red-Green! That means that when you get to the detector, if it is set to one, flash red! Two means red also. But if it's set to three you'd better flash green! Now, move out!"

While Maxwell's demon makes for a flamboyant illustration, there is no such creature to place in Mermin's Box. A more realistic flashing mechanism could be provided by assuming that the particles are identical three-sided objects, like some sort of atomic sized space ships. (Visualize a juice can with a pointed top and a rounded bottom, as illustrated in Figure 2-8.) In this example, having the characteristics of Red-Red-Green means that the pointed top is red, the rounded bottom is red, and the cylindrical body is green.

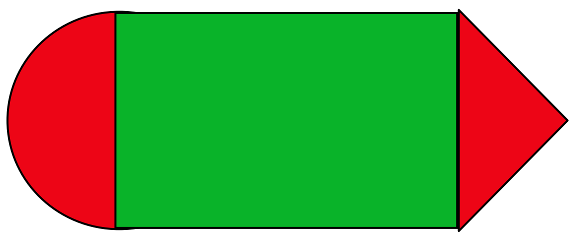


Figure 2-8 The atomic spaceship: one possible way to imagine the hidden variables carried by the particles.

In this second scheme, each detector has three docking ports available. Docking port number one is a cone shaped receptacle. Docking port number two is a half-sphere receptacle. Docking port three consists of a flat space upon which to roll the cylinder. When the detector is set to number one, the doors to docking ports two and three are closed, and only the conical receptacle is available for docking. The particle plugs into that port, and the color of the pointed top is identified and signaled with a flash. If the detector is set to number two, the only open available port is shaped to receive a half sphere. Somehow, the particle maneuvers itself to dock appropriately, the color of the bottom is determined, and that color is communicated with the flashing light. If the detector is set at number three, the only option the particle has is to be rolled along a scanner which notes the color of the side and activates the appropriate colored flash.

2.6 Hidden Variables

In both scenarios, using either Maxwell's demonic gym teacher or the atomic world space port, the salient feature is that the particles carry the information needed to trigger the appropriate response within themselves. Each pair of particles has matching intrinsic characteristics; innate encoded instruction sets that must be obeyed. That these built-in traits exist seems to be the straightforward, obvious conclusion based on Rule One: matching colors for matching detector settings. These innate characteristics are called hidden variables and were Einstein's explanation of choice for the workings of the quantum world. (See footnote, page 7.)

Let's examine the hidden variables we assume each particle carries. Following is a table enumerating the possible codes carried by the particles and the results each code set would produce given the possible detector settings. Again, we stress that since there are three possible detector settings that can be encountered by the particles, there must be a minimum of three bits of information carried by the particles. Since both detectors flash identical colors when set to identical settings, the three bits of information carried by each of the particles must be the same.

Encoded Instruction Sets

Det #1	Det #2	RRR	RRG	RGR	RGG	GRR	GRG	GGR	GGG
1	1	RR	RR	RR	RR	GG	GG	GG	GG
1	2	RR	RR	RG	RG	GR	GR	GG	GG
1	3	RR	RG	RR	RG	GR	GG	GR	GG
2	1	RR	RR	GR	GR	RG	GR	GG	GG
2	2	RR	RR	GG	GG	RR	RR	GG	GG
2	3	RR	RG	GR	GG	RR	RG	GR	GG
3	1	RR	GR	RR	GR	RG	GG	RG	GG
3	2	RR	GR	RG	GG	RR	GR	RG	GG
3	3	RR	GG	GG	GG	RR	GG	RR	GG

Identical settings yield identical flashes 100% of the time

Different settings yield identical flashes 25% of the time

Figure 2-9 The results of each possible hidden variable set, given the settings of the detectors

For example, we see that if the detectors are both set to number one, as illustrated in the first row of the table, hidden variables RRR, RRG, RGR, and RGG all yield dual flashes of RR from the two detectors, since a detector setting of 1 triggers the reading of the first bit of information in the set, while the second and third information bits are superfluous. On the other hand, instruction sets GRR, GRG, GGR and GGG all produce matching green flashes from both detectors, since green is the color in the first position.

Encoded Instruction Sets

Det #1	Det #2	RRR	RRG	RGR	RGG	GRR	GRG	GGR	GGG
1	1	RR	RR	RR	RR	GG	GG	GG	GG
1	2	RR	RR	RG	RG	GR	GR	GG	GG
1	3	RR	RG	RR	RG	GR	GG	GR	GG
2	1	RR	RR	GR	GR	RG	GR	GG	GG
2	2	RR	RR	GG	GG	RR	RR	GG	GG
2	3	RR	RG	GR	GG	RR	RG	GR	GG
3	1	RR	GR	RR	GR	RG	GG	RG	GG
3	2	RR	GR	RG	GG	RR	GR	RG	GG
3	3	RR	GG	GG	GG	RR	GG	RR	GG

Identical settings yield identical flashes ☐ 100% of the time

Different settings yield identical flashes ☐ 25% of the time

Figure 2-10 A detailed example of the results of measurement given that Detector #1 is set to 2, and Detector #2 is set to 1.

For a more complicated example, look at the results from the highlighted row in the table above (Figure 2-10). If Detector #1 is set to position two and Detector #2 is set to position one, an instruction set of RRG yields RR (the first two colors), or makes both detectors flash red, while GGR makes detector #1 flash green while detector #2 flashes red, and so on.

By looking at the three rows enumerating the results for detectors set the same (11, 22 and 33) we see that so far, our hidden variable system is working admirably; identical settings always result in identical responses. We have deduced the least amount of information to be carried by each particle to make our detectors always follow Rule One.

Encoded Instruction Sets

Det #1	Det #2	RRR	RRG	RGR	RGG	GRR	GRG	GGR	GGG
1	1	RR	RR	RR	RR	GG	GG	GG	GG
1	2	RR	RR	RG	RG	GR	GR	GG	GG
1	3	RR	RG	RR	RG	GR	GG	GR	GG
2	1	RR	RR	GR	GR	RG	GR	GG	GG
2	2	RR	RR	GG	GG	RR	RR	GG	GG
2	3	RR	RG	GR	GG	RR	RG	GR	GG
3	1	RR	GR	RR	GR	RG	GG	RG	GG
3	2	RR	GR	RG	GG	RR	GR	RG	GG
3	3	RR	GG	RR	GG	RR	GG	RR	GG

Identical settings yield identical flashes 100% of the time

Figure 2-11 Identical settings always yield identical signals from the detectors

But wait! What about Rule Two? Rule Two states: *When the detectors are turned to different settings, they flash the same color 25% of the time*. Do our hidden variables produce these results? After all, they worked so elegantly to provide the first expected behavior. Are there more insights to be gained?

Consider what happens when the detector switches are set differently. Recall that it is totally acceptable to start with the detectors set the same and then switch one (or both) so that the settings are different after the particles have been sent on their way. This is important to keep in mind, because it suggests that once the particles have exited the source, there is no way to change their hidden variables. Using the hidden variables necessary to produce behavior one, let's see if we get the second observed behavior.

2.7 The Conundrum

Encoded Instruction Sets

Det#1	Det#2	RRR	RRG	RGR	RGG	GRR	GRG	GGR	GGG
1	1	RR	RR	RR	RR	GG	GG	GG	GG
1	2	RR	RR	RG	RG	GR	GR	GG	GG
1	3	RR	RG	RR	RG	GR	GG	GR	GG
2	1	RR	RR	GR	GR	RG	GR	GG	GG
2	2	RR	RR	GG	GG	RR	RR	GG	GG
2	3	RR	RG	GR	GG	RR	RG	GR	GG
3	1	RR	GR	RR	GR	RG	GG	RG	GG
3	2	RR	GR	RG	GG	RR	GR	RG	GG
3	3	RR	GG	GG	GG	RR	GG	RR	GG

Different settings yield identical flashes ☐ 25% ~~of~~ the time

RR = 1/6 GG = 1/6 Total = 1/3

Figure 2-12 Our hidden variables do not give the expected values for identical flashed given that the detectors are set differently.

Herein lies the conundrum. We assumed, based on Rule One that the particles carry with them their instructions governing how to behave upon detection. Although it was not clear whether the hidden variables were intrinsic; hard coded into their very essence like some kind of particle DNA (as illustrated by our three sided particle searching for a place to dock) or whether it was somehow assigned; given to the particles at birth like a manifest destiny tattoo on the forehead (as portrayed by our demonic 7th grade particle gym teacher), the existence of these encoded instructions seemed to be undeniable and absolutely necessary to overcome the logistical problem of not knowing what the detector setting was until arrival. But the logical conclusion based on Rule One is contradicted by Rule Two.

Our hidden variables embodied the minimal instruction sets necessary to force the system to obey Rule One, matching flashes given matching settings. This minimalist set of instructions did not work to produce Rule Two. Perhaps we need another hidden variable in our instruction set. Perhaps a third Rule will do the trick. Somehow, we must either modify the probability space such that it matches both the expectation values for Rule One and for Rule Two, or the hidden variables themselves must change to provide the expected quantum statistical results.

2.8 Inside Mermin's Box

We have already divulged the quantum secret of Mermin's box. Here are the details. Inside the source is an emitter of spin-½ particles in the singlet state, while the detectors are actually Stern-Gerlach magnets which set up a magnetic field. The magnets can be oriented vertically in the plane perpendicular to the flight path, or at $\pm 180^{\circ}$ from the vertical, with the angle θ denoting the difference between the two orientations. The detectors measure whether the spin of the particle is in the direction of the field or opposite the field. There is a catch. The magnets are oriented the same ($\theta = 0$) when the detector settings are the same. But since one particle is spin up and the other spin down, this would result in one red flash and one green flash. The trick is that the detectors have opposite signaling protocols. One detector is set to flash red if the particle's spin is found to be in the direction of the field and green if its spin is in the opposite direction. The second detector must flash green if the particle's spin is in the direction of the field and red if oriented opposite the field.

As for the case where the magnets (detector settings) are different, Mermin [13] states:

> *"It is a well-known elementary result that when the orientations of the magnets differ by an angle θ, then the probability of spin*

measurements of each particle yielding opposite values is $\cos^2(\theta/2)$. *This probability is unity when* $\theta = 0$ *and 1/4 when* $\theta = \pm 120°$.*"*

Now that we understand Mermin's Box, inside and out, there is one more necessary ingredient to be identified before we can build our own mechanical and deterministic, yet quantum box. We must have some way to test whether or not our object is behaving in a quantum manner. For that we turn to Bell's Theorem.

Chapter 3

Bell's Theorem

As discussed in Chap. 1, Bell's Theorem shows that if one has a system with *local* hidden variables, joint measurements across three of those variables will *always* obey Bell's Inequality. Conversely, Bell's Inequality will be violated for *some* values of these same joint measurements if the system is nonlocal. Since quantum systems also exhibit nonlocal tendencies, this property makes Bell's Inequality an ideal test equation for both quantum attributes in general and nonlocality specifically.

3.1 Bell's Inequality

Here is Bell's Inequality [10]:

$$\left| E_{HV}\left(\hat{a},\hat{b}\right) - E_{HV}\left(\hat{a},\hat{c}\right) \right| \leq 1 + E_{HV}\left(\hat{b},\hat{c}\right). \tag{3.1}$$

It states specifically that for a system with ***local hidden variables***, upon which three binary measurements can be made, the absolute value of the difference in expectation values for the product of the first measurement with each of two other measurements must be less than or equal to unity plus the expectation value of the product of the second and third measurements together.

In other words, Bell's Inequality is a comparison of the correlations between three different measurements on the same system. These three measurements are binary. A binary measurement is one that returns only one of two possible results; +1 or -1, red or green, black or white, etc. In this case, the only possible values are +1 or -1, and therefore the only conceivable values for their correlations[7] are the products +1 or -1. A three-way comparison is made; the first measurement $\hat{a}$ with the second measurement $\hat{b}$ yields the correlation $\hat{a}\hat{b}$, $\hat{b}$ with $\hat{c}$ returns $\hat{b}\hat{c}$ and the only other possible combination, $\hat{a}$ and $\hat{c}$, produces $\hat{a}\hat{c}$.

For Bell's Theorem to be useful in the reality of the working experimental lab, another option must be added to the binary ± 1: the null reading. We will see in section 3.5 that the tacit acknowledgement that experimental apparatus don't always function perfectly results in the softening of the equality to an inequality.

3.2 Proof of Bell's Inequality

George Greenstein and Arthur Zajonc, in their book The Quantum Challenge [19], provide the following proof that any local hidden variable theory will obey Bell's inequality:

In this proof we will use the two distinct observers, Alice and Bob, and call the results of their measurements respectively A and B. If a theory has hidden variables (λ)

[7] In *The Quantum Challenge,* the authors use the term "expectation value" when discussing the joint correlation between two measurements used by Bell in his inequality. Ballentine uses the word "correlation" and Bell calls the same quantity a "probability". In all three cases, the intended meaning is the joint behavior of the measured variables. We have used the term "expectation value" in our discussion of both Bell's Inequalities and we have changed the author's original notation in the two Bell's Inequality proofs so that the same convention is used consistently throughout this thesis. We will continue with this usage while discussing the measurements in our Q Box of Chap. 4 and the computer simulation described in Chap. 5.

and is local (the two givens in Bell's Theorem), then the measurement A can only depend on the angle θ between the hidden variable λ and $\hat{a}$, the angle at which Alice's analyzer is set. More specifically, the measurement A cannot have any correlation with B, nor can Bob in any way influence what is measured by Alice. The variable λ is assumed to be a sort of local catchall of all of the unknowns in the system. Mathematically, we can say that A is a function of λ and $\hat{a}$, and that B is a function of λ and $\hat{b}$, with no cross correlations:

$$\begin{aligned} A &= A(\hat{a},\lambda) \quad \textit{not} \quad A(\hat{a},\hat{b},\lambda) \\ B &= B(\hat{b},\lambda) \quad \textit{not} \quad B(\hat{a},\hat{b},\lambda). \end{aligned} \tag{3.2}$$

The expectation value of AB is the average of AB over all possible values of the hidden variable λ. This is a general statement, valid for any hidden variable theory:

$$E_{HV}\left(\hat{a},\hat{b}\right) = \int AB d\lambda. \tag{3.3}$$

The assumption that we have enumerated all possible values of λ leads to

$$\int d\lambda = 1. \tag{3.4}$$

If the particles are entangled such that they have opposite values, then having Alice and Bob set their analyzers parallel to each other they will also yield opposite values:

$$A(\hat{a},\lambda) = -B(\hat{a},\lambda). \tag{3.5}$$

Inserting these expressions into Bell's Inequality, we see that

$$\begin{aligned} E_{HV}(\hat{a},\hat{b}) - E_{HV}(\hat{a},\hat{c}) &= \int \left[A(\hat{a},\lambda)B(\hat{b},\lambda) - A(\hat{a},\lambda)B(\hat{c},\lambda)\right] d\lambda \\ &= -\int \left[A(\hat{a},\lambda)A(\hat{b},\lambda) - A(\hat{a},\lambda)A(\hat{c},\lambda)\right] d\lambda. \end{aligned} \tag{3.6}$$

Since A and B must be either one or negative one, it follows that

$$\left[A(\hat{b},\lambda)\right]^2 = 1, \tag{3.7}$$

and therefore

$$E_{HV}(\hat{a},\hat{b}) - E_{HV}(\hat{a},\hat{c}) = -\int A(\hat{a},\lambda)A(\hat{b},\lambda)\left(1 - A(\hat{b},\lambda)A(\hat{c},\lambda)\right) d\lambda. \tag{3.8}$$

By the same argument, $A(\hat{a},\lambda)A(\hat{b},\lambda)$ also equals ± 1, and must be less than or equal to $+1$. Eliminating this term from Equation (3.8) and taking the absolute value gives us:

$$\left|E_{HV}(\hat{a},\hat{b}) - E_{HV}(\hat{a},\hat{c})\right| \le \left|\int \left(1 - A(\hat{b},\lambda)A(\hat{c},\lambda)\right)\right| d\lambda \tag{3.9}$$

which can be rewritten as

$$
\begin{aligned}
\left|E_{HV}\left(\hat{a},\hat{b}\right)-E_{HV}\left(\hat{a},\hat{c}\right)\right| &\leq 1+\int A\left(\hat{b},\lambda\right)B\left(\hat{c},\lambda\right)d\lambda \\
&\leq 1+E_{HV}\left(\hat{b},\hat{c}\right)
\end{aligned}
\tag{3.10}
$$

This is the version of Bell's Theorem that we will be using to evaluate our deterministic Q Box in Chap. 5.

3.3 Bell's Inequality in the Lab

In quantum optics or quantum spin experiments, the three measurements are made on either two entangled photons or two entangled spin-½ particles. The experimenters can be arbitrary distances from each other and from the source. Traditionally, the two scientists making the measurements are named Alice and Bob.

To start the measurement process, Alice sets her analyzer at some angle $\hat{a}$ to her beam of light or particle trajectory. Bob, meanwhile, rotates his analyzer to some other angle $\hat{b}$ relative to his light beam or particle flight path. Each of them makes some kind of binary measurement; yes or no, up or down, parallel or perpendicular to the axis. The results are recorded. Next Bob changes the setting on his analyzer to a new orientation, $\hat{c}$. Alice leaves her analyzer in the original $\hat{a}$ position and again they each make a record of what they see. For the third measurement set, Alice changes her analyzer to the $\hat{b}$ orientation Bob used in the first measurement, while this time Bob leaves his analyzer untouched and again measures $\hat{c}$.

The measurement and recording process is repeated over and over. When enough data has been gathered to make the data collection statistically viable, the expectation values of each of the three aggregate measurements are calculated and compared in Bell's Inequality.

For example, the experimental value of $E_{\exp}\left(\hat{a},\hat{b}\right)$ is found by checking the data recording sheets and compiling the results of all of the paired detection events for which Alice's detector was set at $\hat{a}$ and Bob's was set at $\hat{b}$, as follows:

$$E_{\exp}\left(\hat{a},\hat{b}\right)=\frac{N_{++}+N_{--}-N_{+-}-N_{-+}}{N}. \qquad (3.11)$$

N_{++} is the number of data events where Alice's detector was set at $\hat{a}$, Bob's was set at $\hat{b}$, and both detectors fired +1, N_{--} is the number of data events where both detectors fired -1, N_{+-} denotes detector 1 (Alice) measuring +1 and detector 2 (Bob) measuring -1, and finally, N_{-+} represents the situation where Alice encounters -1 and Bob gets a reading of +1. N represents the total number of pairs emitted from the source while Alice and Bob were set to $\hat{a}$ and $\hat{b}$, respectively, and includes all of the misfires: N_{+0}, N_{-0}, N_{0+}, N_{0-} and N_{00}. It is impossible to determine the exact number of pairs emitted where both escaped detection (the N_{00} case), but making the assumption that the particles that *were* detected are a valid statistical sample of the whole, we can say that

$$E_{\exp}\left(\hat{a},\hat{b}\right)=\frac{N_{++}+N_{--}-N_{+-}-N_{-+}}{N_{++}+N_{--}+N_{+-}+N_{-+}}. \qquad (3.12)$$

It must be emphasized that for some possible settings on the analyzers, the theoretical predictions of quantum mechanics conform to Bell's Inequality; in other words, some analyzer configurations produce results consistent with both classical and quantum statistics. However, for other combinations of settings, Bell's Inequality *is* violated, and the system is manifestly nonlocal.

Let's examine more closely one way in which the actual measurements are made, and which experimental situations produce classical outcomes and which give quantum outcomes in one of the most popular types of Bell's Inequality experiments: a Stern-Gerlach experiment with spin-½ particles.

3.4 EPR Stern-Gerlach Experiments with spin-½ particles

In an EPR Stern-Gerlach experiment, a source emits two entangled particles in different directions and with opposite spins. Each particle is sent between two magnets which are producing a nonuniform magnetic field. The magnets act to sort the particles based on their spin by deflecting them to different positions on a screen. A measurement is made of the spin component along the arbitrary axis chosen by the scientist. The axes chosen become the $\hat{a}$, $\hat{b}$ and $\hat{c}$ measurement protocols, with spin up having the value of (+1) and spin down having the value of (-1) along the corresponding axis.

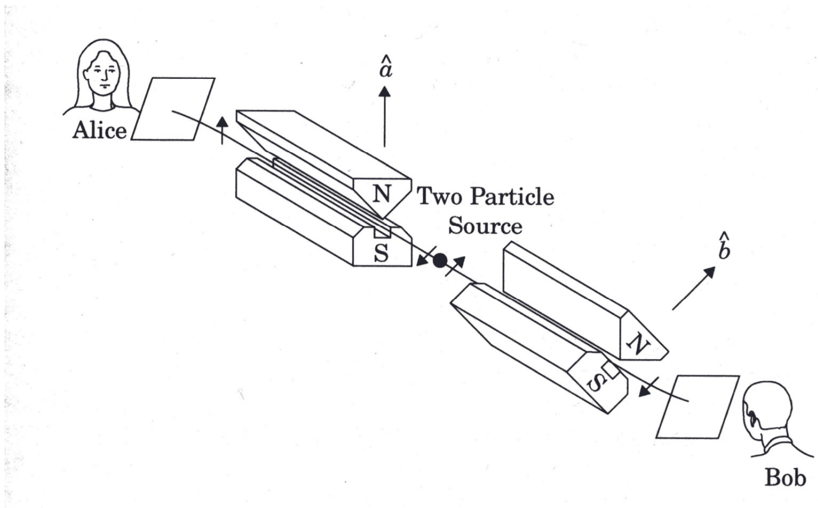


Figure 3-1 An illustration of an EPR Stern-Gerlach experimental setup. Note that Alice's and Bob's analyzers are oriented in the $\hat{a}$ and $\hat{b}$ directions. [Reprinted from *The Quantum Challenge. p. 117]*

Quantum mechanics predicts that for an EPR Stern-Gerlach experimental arrangement, the expectation value of a joint measurement between Alice's $\hat{a}$ and Bob's $\hat{b}$ is

$$E_{QM}\left(\hat{a}\hat{b}\right) = -\hat{a}\cdot\hat{b} = -\cos\theta\,, \tag{3.13}$$

with θ as the angle between the $\hat{a}$ and $\hat{b}$ settings.

On the other hand, for a hidden variable theory, the expectation value based on θ is

$$E_{HV}\left(\hat{a}\hat{b}\right) = \frac{2}{\pi}\theta - 1. \tag{3.14}$$

Both equations yield the same results for some values of θ. Following is an illustration showing where the two equations are and are not equivalent. Figure 3-2 shows that for $\theta = 0, \pi/2, and\ \pi$, the predictions based on a hidden variable theory match those expected from quantum mechanics, while an orientation of 22.5° between analyzers produces a strong violation of Bell's Inequality [20].

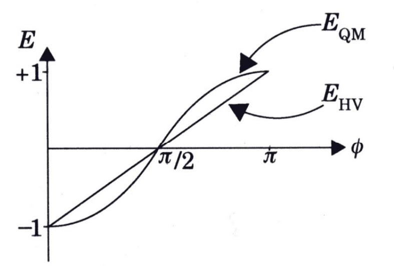


Figure 3-2 Predictions for the expectation values of quantum mechanical and hidden variable systems. [Reprinted from *The Quantum Challenge. p. 121]*

3.5 An Alternative Bell's Inequality, BI2

In 1971, Bell put forth a second inequality [21]:

$$\left|E_{HV}\left(\hat{a},\hat{b}\right) - E_{HV}\left(\hat{a},\hat{b}'\right)\right| + \left|E_{HV}\left(\hat{a}',\hat{b}'\right) + E_{HV}\left(\hat{a}',\hat{b}\right)\right| \leq 2. \tag{3.15}$$

This inequality, which we will call Bell's Second Inequality, or BI2 for short, was actually first derived by Clauser, Holt, Horne and Shimony [22] in 1969, in an attempt to find a Bell's- type inequality that could be tested experimentally. BI2 was designed specifically for experiments using spin-½ particles but can be generalized to any system having two binary components measured by two devices. It allows for a different experimental configuration than the original Bell's Inequality that we previously discussed and will be useful for analyzing Mermin's Box, so we will provide a short discussion and proof.

In this second incarnation of Bell's Inequality, (or BI2), each measuring device is assumed to have a range of possible orientations corresponding to the possible configurations in a Stern-Gerlach apparatus. The correlations between four combinations of settings made from two possible settings on each detector are compared. The first detector contributes $\hat{a}$ and $\hat{a}'$, and the second detector provides measurements $\hat{b}$ and $\hat{b}'$. This differs from Bell's first Inequality from his 1964 paper (and discussed in Sec. 3.2) where there are three sets of measurements taken:

$$E_{HV}\left(\hat{a},\hat{b}\right),\ E_{HV}\left(\hat{a},\hat{c}\right) \text{ and } E_{HV}\left(\hat{b},\hat{c}\right). \tag{3.16}$$

The following proof is from *Quantum Mechanics*, by Leslie Ballentine [23]. The derivation follows very closely that of the original Bell's Inequality proof provided in Sec. 3.2. Like its predecessor, BI2 demonstrates that any local hidden variable theory will obey an inequality:

$$\left|E_{HV}\left(\hat{a},\hat{b}\right)-E_{HV}\left(\hat{a},\hat{b}'\right)\right|+\left|E_{HV}\left(\hat{a}',\hat{b}'\right)+E_{HV}\left(\hat{a}',\hat{b}\right)\right|\leq 2. \tag{3.17}$$

For BI2, the first device (Alice's) is initially set at $\hat{a}$, and Bob's device starts with a $\hat{b}$ orientation. The only controllable factors in the measurements are assumed to be the detector settings, and all other uncontrollable factors are lumped together and labeled λ. Therefore, the two measurement outcomes can be described mathematically as:

$$\begin{aligned} &\mathrm{A}(\hat{a},\lambda) = \pm 1 \\ &\mathrm{B}(\hat{b},\lambda) = \pm 1 \end{aligned} \tag{3.18}$$

In order to accommodate null measurements when either one or both of the detectors fail, the binary measurement restrictions Eq. (3.18) are loosened to inequalities:

$$\begin{aligned} &|A(a,\lambda)| \leq 1 \\ &|B(b,\lambda)| \leq 1 \end{aligned} \tag{3.19}$$

As in Eq. (3.2), the locality assumption disallows any cross-correlations and makes the two measurements independent of each other. The catchall parameter λ is also manifestly local, with the assumption that these uncontrollable parameters follow some probability distribution $\rho(\lambda)$ where $\rho(\lambda) \geq 0$ and $\int \rho(\lambda) d\lambda = 1$. The expectation value (or correlation, or probability) of a joint measurement across the two detectors is

$$E_{HV}(\hat{a},\hat{b}) = \int A(\hat{a},\lambda) B(\hat{b},\lambda) \rho(\lambda) d\lambda. \tag{3.20}$$

We examine the correlation results using the four possible settings: $\hat{a}$ and $\hat{a}'$ (Alice) on the first detector and $\hat{b}$ and $\hat{b}'$ on the second detector (Bob). Using Eq. (3.20), we find:

$$
\begin{aligned}
E_{HV}\left(\hat{a},\hat{b}\right) - E_{HV}\left(\hat{a},\hat{b}'\right) &= \int \left[A(\hat{a},\lambda)B\left(\hat{b},\lambda\right) - A(\hat{a},\lambda)B\left(\hat{b}',\lambda\right)\right]\rho(\lambda)d\lambda \\
&= \int \left[A(\hat{a},\lambda)B\left(\hat{b},\lambda\right)\left(1 \pm A(\hat{a}',\lambda)B\left(\hat{b}',\lambda\right)\right)\right]\rho(\lambda)d\lambda \\
&- \int \left[A(\hat{a},\lambda)B\left(\hat{b}',\lambda\right)\left(1 \pm A(\hat{a}',\lambda)B\left(\hat{b},\lambda\right)\right)\right]\rho(\lambda)d\lambda.
\end{aligned} \tag{3.21}
$$

Now, by using the inequalities from Eq. (3.19), we obtain the relationship:

$$
\begin{aligned}
\left|E_{HV}\left(\hat{a},\hat{b}\right) - E_{HV}\left(\hat{a},\hat{b}'\right)\right| &\leq \int \left[1 \pm A(\hat{a}',\lambda)B\left(\hat{b}',\lambda\right)\right]\rho(\lambda)d\lambda \\
&+ \int \left[1 \pm A(\hat{a}',\lambda)B\left(\hat{b},\lambda\right)\right]\rho(\lambda)d\lambda \\
&= 2 \pm \left[E_{HV}\left(\hat{a}',\hat{b}'\right) - E_{HV}\left(\hat{a}',\hat{b}\right)\right]
\end{aligned} \tag{3.22}
$$

which implies Bell's second Inequality, or BI2:

$$
\left|E_{HV}\left(\hat{a},\hat{b}\right) - E_{HV}\left(\hat{a},\hat{b}'\right)\right| + \left|E_{HV}\left(\hat{a}',\hat{b}'\right) + E_{HV}\left(\hat{a}',\hat{b}\right)\right| \leq 2. \tag{3.23}
$$

To show how this inequality is violated for a quantum system, we will compare Eq. (3.23) with the quantum mechanical correlation for the spin-½ system. If $\sigma_a \equiv \hat{\sigma} \bullet \hat{a}$ and $\sigma_b \equiv \hat{\sigma} \bullet \hat{b}$ represent the Pauli spin operator components in the directions of the unit vectors $\hat{a}$ and $\hat{b}$, then a measurement of the spin of particle 1 along the direction $\hat{a}$ will be correlated with a measurement of the spin of the second particle along $\hat{b}$. The degree of quantum correlation is a function of the angle between $\hat{a}$ and $\hat{b}$:

$$
\left\langle \Psi_0 \left| \sigma_a \otimes \sigma_b \right| \Psi_0 \right\rangle = -\cos(\theta_{ab}). \tag{3.24}
$$

In other words, the *quantum expectation value* as a function of the angle between the two settings is:

$$E_{QM}(\theta) = -\cos(\theta), \tag{3.25}$$

whereas the *BI2* expectation value as a function of the angle θ between the two settings is:

$$E_{BI2}(\hat{a}, \hat{b}) = E_{BI2}(\theta_{ab}). \tag{3.26}$$

We will compare these two predictive results for the experimental setup shown below...

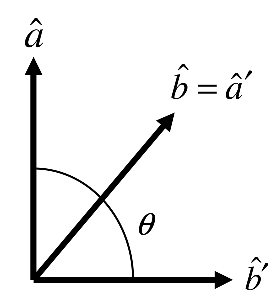


Figure 3-3 Detector settings to test BI2 against the predictions of quantum mechanics. [Reproduced from Ballentine's *Quantum Mechanics. p. 443*]

In the above configuration we have assumed that the instrument settings are coplanar, and that the angle θ between $\vec{a}$ and $\vec{a}'$ is the same as the angle between $\vec{b}$ and $\vec{b}'$. For this particular experimental arrangement, the prediction for BI2 becomes

$$|E_{BI2}(\theta) - E_{BI2}(2\theta)| + |E_{BI2}(\theta) - E_{BI2}(0)| \leq 2. \qquad (3.27)$$

By inserting the quantum expectation values (Eq. 3.25) into the inequality above, we obtain

$$2\cos(\theta) - \cos(2\theta) \leq 1, \qquad (3.28)$$

which is violated for many values of θ. For example, $\pi/3$, the angle of maximum violation gives:

$$2\cos(\pi/3) - \cos(2\pi/3) = 3/2. \qquad (3.29)$$

Since 3/2 is certainly not less than or equal to one, the quantum expectation values violate BI2. As Ballentine states [23], "Therefore quantum mechanics is in conflict with at least one of the assumptions that were used in the derivation of Bell's Inequality." Again, these assumptions are locality, hidden variables and realism.[8]

Note that in BI2, the two assumptions were hidden variables, the attributes unknown but measurable by their instruments, and locality, in that the measurements made by Alice can in no way effect the measurements made by Bob. All other possible

[8] Both Ballantyne and Stapp dismiss reality issues as unproductive lines of inquiry. Stapp because one need not make assumptions about the nature of reality in the derivation of Bell's theorem and Ballantyne because it is a "philosophical view. . . It is not clear what the critics of realism and objectivism would offer as alternatives ("idealism" and subjectivism"?), nor how that would be useful in understanding quantum mechanics. [23] [18]

effects are contained in λ, which is also expressly local. Therefore BI2 is a test for nonlocality in systems with hidden variables.

As an illustration of how Bell's Inequality works to highlight quantum-like statistical inconsistencies, we will use BI2 to analyze Mermin's Box.

3.6 A Bell's Inequality Analysis of Mermin's Box

Recall from chapter 2 that Mermin's Box has one source box emitting two particles. One particle goes to one detector while the other particle ends up at the second detector. The detectors can be set to one of three distinct settings. These detectors exhibit two behaviors which we enumerate as follows:

> ***Rule One***: *Every time the detectors are identically set they flash the same color, although* ***which*** *color is not specified.*
>
> ***Rule Two***: *When the detectors are turned to different settings, they flash the same color about 25% of the time.*

The detectors always flash the same color, either red or green, if the detectors are tuned to the same setting, and the detectors can be arbitrarily far apart and their settings changed after the particles are in flight. Therefore, it is assumed that the particles must carry from the time of their inception three item information sets (hidden variables) that either 'tells' the particles how to respond to each of the three detector settings, or that are recognizable in some way by the detector so that it can make a decision on how to respond.

Encoded Instruction Sets

Det #1	Det #2	RRR	RRG	RGR	RGG	GRR	GRG	GGR	GGG
1	1	RR	RR	RR	RR	GG	GG	GG	GG
1	2	RR	RR	RG	RG	GR	GR	GG	GG
1	3	RR	RG	RR	RG	GR	GG	GR	GG
2	1	RR	RR	GR	GR	RG	GR	GG	GG
2	2	RR	RR	GG	GG	RR	RR	GG	GG
2	3	RR	RG	GR	GG	RR	RG	GR	GG
3	1	RR	GR	RR	GR	RG	GG	RG	GG
3	2	RR	GR	RG	GG	RR	GR	RG	GG
3	3	RR	GG	GG	GG	RR	GG	RR	GG

Identical settings yield identical flashes ☐ 100% of the time

Different settings yield identical flashes ☐ 25% of the time

Figure 3-4 The results of each possible hidden variable set, given the settings of the detectors.

Above is a table summarizing the possible detector settings, possible hidden variables and the resulting detector responses. In order to use BI2

$$\left|E_{HV}\left(\hat{a},\hat{b}\right)-E_{HV}\left(\hat{a},\hat{b}'\right)\right|+\left|E_{HV}\left(\hat{a}',\hat{b}'\right)+E_{HV}\left(\hat{a}',\hat{b}\right)\right|\leq 2 \qquad (3.30)$$

to analyze the system we will need to decide which detector settings will serve as $\hat{a}$, $\hat{a}'$, $\hat{b}$ and $\hat{b}'$. In Sec. 3.5 we found that the inequality is most strongly violated when $\hat{b}=\hat{a}'$ and the angles between $\hat{a}$ and $\hat{a}'$ on the first detector and $\hat{b}$ and $\hat{b}'$ on the second detector are the same. Since Mermin's is a 'black' box where we are not supposed to know what is inside, or how the inner workings might correspond to polarization angles, we will use our best judgment in deciding how to model this angular requirement. To be consistent with the description above we will use $\hat{a}$=1, $\hat{b}$=2, $\hat{a}'$=2 and $\hat{b}'$=3 as our first settings, although these are completely arbitrary and have no connection to any known measurement. Given these values for our detector settings we use the highlighted rows of our table to calculate the expectation values for BI2.

Alice	Bob	RRR	RRG	RGR	RGG	GRR	GRG	GGR	GGG
1	1	RR	RR	RR	RR	GG	GG	GG	GG
1	2	RR	RR	RG	RG	GR	GR	GG	GG
1	3	RR	RG	RR	RG	GR	GG	GR	GG
2	1	RR	RR	GR	GR	RG	GR	GG	GG
2	2	RR	RR	GG	GG	RR	RR	GG	GG
2	3	RR	RG	GR	GG	RR	RG	GR	GG
3	1	RR	GR	RR	GR	RG	GG	RG	GG
3	2	RR	GR	RG	GG	RR	GR	RG	GG
3	3	RR	GG	GG	GG	RR	GG	RR	GG

Figure 3-5 The results of each possible hidden variable set, given the settings of the detectors. The light blue row shows the results of ($\hat{a}$, $\hat{b}$), the pink row is ($\hat{a}$, $\hat{b}'$), the yellow row ($\hat{a}'$, $\hat{b}$) and the green row ($\hat{a}'$, $\hat{b}'$)

The light blue row corresponds to the joint measurement ($\hat{a}$, $\hat{b}$), the pink row details ($\hat{a}$, $\hat{b}'$), the yellow row tabulates ($\hat{a}'$, $\hat{b}$) and the green row represents ($\hat{a}'$, $\hat{b}'$). Mermin doesn't give us any information about the efficiency of his Gedankendetectors, so we will assume that they work with "Gedankenperfection". The measurement values must be binary, so we will assign the value of (+1) to red and the value of (-1) to green. Because of the inherent symmetry in the system, the row itemizing the results for the ($\hat{a}$, $\hat{b}$) measurement results has an expectation value of zero.

Alice	Bob	RRR	RRG	RGR	RGG	GRR	GRG	GGR	GGG
1	1	RR	RR	RR	RR	GG	GG	GG	GG
1	3	RR	RG	RR	RG	GR	GG	GR	GG
2	1	RR	RR	GR	GR	RG	GR	GG	GG
2	2	RR	RR	GG	GG	RR	RR	GG	GG
2	3	RR	RG	GR	GG	RR	RG	GR	GG
3	1	RR	GR	RR	GR	RG	GG	RG	GG
3	2	RR	GR	RG	GG	RR	GR	RG	GG
3	3	RR	GG	GG	GG	RR	GG	RR	GG

1	2	RR	RR	RG	RG	GR	GR	GG	GG

1	2	(-1)(-1)=1	(-1)(-1)=1	(-1)(+1)=-1	(-1)(+1)=-1	(+1)(-1)=-1	(-1)(+1)=-1	(+1)(+1)=1	

1	2	1	1	-1	-1	-1	-1	1	1

$$\boxed{E_{Mer} = \frac{1+1-1-1-1-1+1+1}{8} = 0}$$

Figure 3-6 The calculation of the expectation values for the $(\hat{a}, \hat{b})$ measurement of Mermin's Box.

The values for the other three expectation value are determined in the same way.

$$E_{Mer}\left(\hat{a}, \hat{b}'\right) = 0$$
$$E_{Mer}\left(\hat{a}', \hat{b}\right) = 1 \qquad (3.31)$$
$$E_{Mer}\left(\hat{a}', \hat{b}'\right) = 0$$

Inserting these expectation values into BI2 we find that the inequality is satisfied for this combination of measurement values.

$$|0-0|+|0+1|\leq 2 \tag{3.32}$$

There are several combinations of expectation values that work to violate BI2. For example, if $E_{Mer}(\hat{a},\hat{b})=(-1)$ and $E_{Mer}(\hat{a},\hat{b}')=E_{Mer}(\hat{a}',\hat{b}')=E_{Mer}(\hat{a}',\hat{b})=1$, BI2 yields $|(-1)-1|+|1+1|=4$. Unfortunately, symmetry limits the expectation values from Mermin's Box to either +1 or zero; an expectation value of (-1) is not possible. Under these constraints, a close examination of BI2 shows that to violate the inequality one must have the conditions $E_{Mer}(\hat{a},\hat{b})=E_{Mer}(\hat{a}',\hat{b}')=E_{Mer}(\hat{a}',\hat{b})=1$ and $E_{Mer}(\hat{a},\hat{b}')=0$. Upon further investigation, it can be seen from Fig. 3.5 that the only combinations of detector settings that contribute +1 to the inequality are those from identical settings; (1, 1), (2, 2) and (3, 3). All other setting combinations yield 0 because of the symmetry of the system. With only two possible settings for Alice to use: $\hat{a}$ and $\hat{a}'$, and two measurement options for Bob, $\hat{b}$ and $\hat{b}'$, there is no way to construct the three identical pairs required for BI2 violation. Careful study of the table shows that BI2 can *never* be violated, even if we drop our initial assumption that $\hat{b}=\hat{a}'$.

What about the original Bell's inequality? After all, it had three variables to choose from:

$$\left|E_{HV}(\hat{a},\hat{b})-E_{HV}(\hat{a},\hat{c})\right|\leq 1+E_{HV}(\hat{b},\hat{c}) \tag{3.33}$$

A similar analysis to that offered above for BI2 shows that even with three variables from which to construct the expectation values, this inequality can also never be violated. In the case of the original Bell's Inequality, it would require one or more of the expectation values to be negative. Again, the symmetry of the system only generates two expectation values: +1 for the cases of matching detector settings or zero for all other combinations. Therefore, both versions of Bell's Inequality require some sort of measurement mechanism to break the symmetry in the system.

It must be noted that Bell's Inequality is quite general. It proves that it is impossible for the joint expectation values from any system with local hidden variables to match the joint expectation values from any nonlocal system for *all* values of $\hat{a}, \hat{b}$ and $\hat{c}$. It can be reformulated for other configurations and numbers of variables, as evidenced with BI2 by Clauser, Holte, Horne and Shimony. [10] The proof is independent of the particular hidden variable theory. In short, it shows that the statistics for *any* local hidden variable theory will not match those expected from a quantum system. Bell's Theorem was long thought to rule out the possibility of hidden variables, but instead it provides a loophole: hidden variables may be present if they are *nonlocal*.

In Chap. 5 we will address these issues and show that a deterministic system with hidden variables *can* be constructed for which it is possible to violate Bell's Inequality; but it will take a nonlocal measurement scheme to break the symmetry and produce quantum correlations. Before we tackle a nonlocal measurement though, we would like to investigate in Chap. 4 the local measurement strategies used in the Game of Life. Understanding the manner in which individual time steps are evaluated in the Game of Life will be helpful later when we address similar measurement issues in our Q Box.

Chapter 4

The Game of Life

Before continuing with our Q Box measurement scheme, it will be helpful to take a short side trip into the world of cellular automata. [24] Cellular automata, like the Game of Life which we will be discussing, are composed of a grid system of dark and light colored squares which "evolve" according to the application of simple iterative rules. Scientists and mathematicians study the Game of Life and other cellular automata to gain insights into simple, rule-obeying systems that occur in nature; such as human and computer viruses, insect behavior, traffic flow and biological mutations. For example, John von Neumann, one of the pioneers of automaton modeling, used a cellular system in the design of his universal constructor, a theoretical self-replicating machine. [25]

The Game of Life was invented in 1970 by the mathematician John Conway. [24] It is played on a grid like a board for checkers or Go. Each space in the grid is a cell that is either alive (has a marker in it) or dead (empty). The cells live or die based on the state of their eight nearest neighbors: the four cells that share boundary walls with it on each side, and the four cells that touch it at the four corners. One starts the game by laying out some pattern of markers on the grid. The cells are born, remain alive or die based on how many living or dead immediate neighbors they have. The rules for living and dying are:

1) ***Birth:*** *If a dead cell has exactly three live neighbors it becomes alive.*

*2) **Survival:** A living cell with either two or three living neighbors remains alive.*

*3) **Death from loneliness:** A living cell with zero or one living neighbor dies.*

*4) **Death from overcrowding:** A living cell with more than three live neighbors dies.*

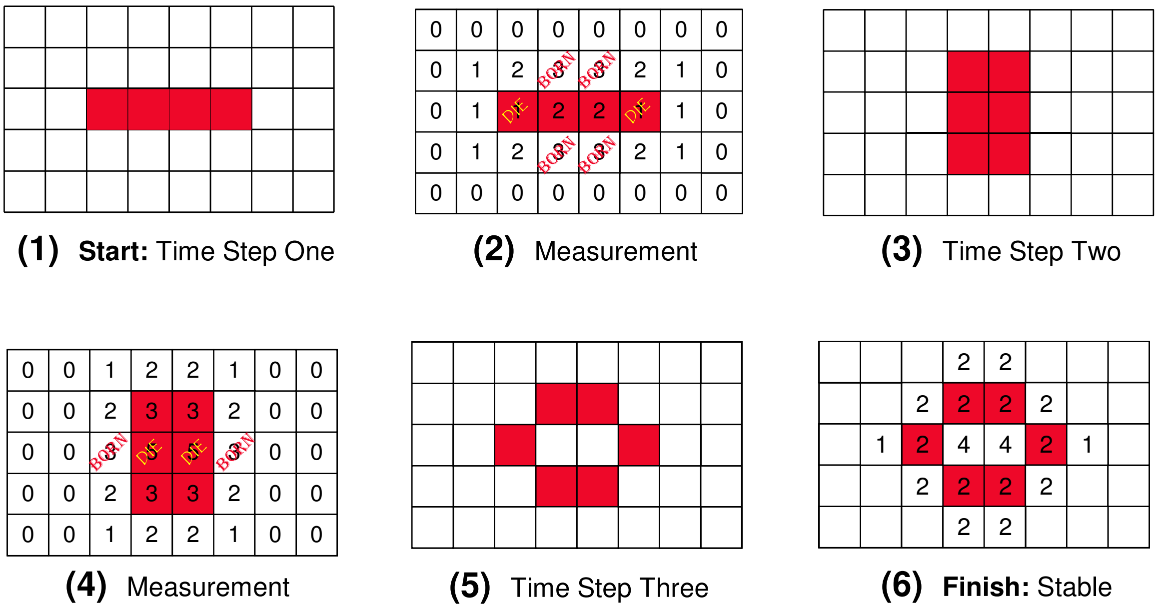


Figure 4-1 Results in the Game of Life Measurement [24]

Above in Figure 4-1 is an example of the evolution of a simple automaton system from its beginning to its stable end. It starts as shown in (1) with four living cells all in a row in the center of a neighborhood consisting of forty cells. Each cell is analyzed separately to ascertain how many adjacent living cells there are surrounding it, as shown in illustrations (2), (4) and (6). Then, if a living cell has either two or three neighbors, it survives in the next time step, as do the two center living cells in (2). If a living cell has more than three living neighbors it dies in the next time step. This happens in (4) to the cells at the center. If a living cell has zero or one living neighbors, it dies in the next time step, as shown by the cells at each end in (2). If a dead cell has exactly three living neighbors it becomes live in the next time step, as illustrated in both (2) and (4). Otherwise, the cells remain dead.

Based on just these simple rules, the patterns in the Game of Life can grow, stabilize or completely die out. Depending on the initial conditions, successive generations of cells living and dying form patterns that can oscillate, pulse, glide across the board, dance like a queen bee, form puffs that grow until they hit a border, or become guns that destroy everything in front of them. They can even replicate themselves.

Here is another simple automaton, made up of five living cells, called a "glider". The five cells from which a glider is constructed mutate through four different repeating patterns. With every cycle the glider moves up the grid.

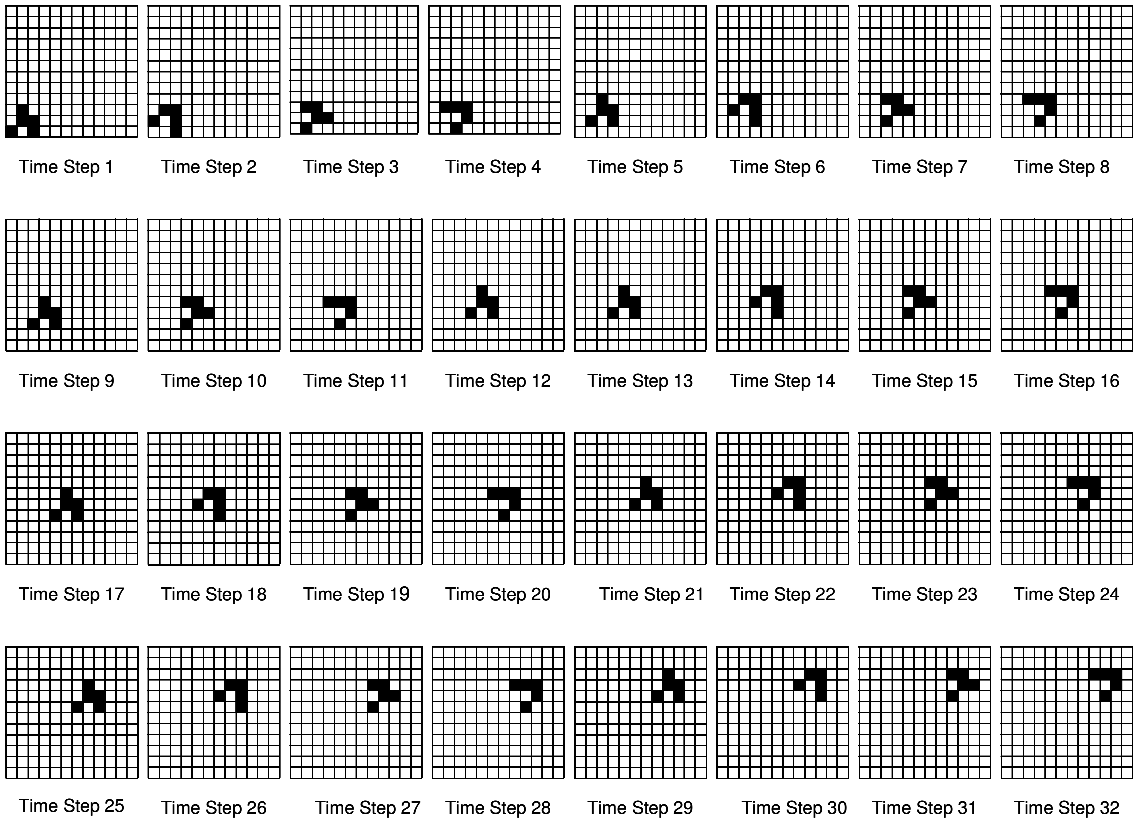


Figure 4-2 A Game of Life glider "moving". [24]

Game of Life simulations are relevant to our research because of the order that the measurement process follows. For each time step a state is decided. Everything stops while cells are measured and those that are to be changed are identified. Every cell is considered separately, and the future of each cell depends only on a joint measurement of its immediate neighbors in the current time step only. For example, if a dead cell in time step 1 has three living neighbors, the cell will be alive in time step 2, even if the those neighboring cells that helped the cell to meet the requirement for birth themselves die in time step 2. Although Game of Life Simulations appear to move with rippling effects across the grid, as if one change triggers another, in actuality the life or death decisions are all based on the end state of the previous time step. Each time step is discrete.[9]

Our measurement process will follow this same pattern. In the Game of Life the cells live or die in the next time step based on a joint measurement of their immediate neighbors in the current time step. We will also make a joint measurement which will determine the future state of the variables of our Q Box. There is a distinct difference, however, in how one plays the Game of Life and how one plays with the Q Box; in our Q Box the joint measurement is ***nonlocal.***

[9] For an excellent tutorial and fascinating examples, we highly recommend http://www.math.com/students/wonders/life/life.html, March 2, 2006. [26]

Chapter 5

A Mechanically Classical, Yet Statistically Quantum Box

Having investigated the inherent peculiarities of quantum systems as illustrated by David Mermin's Box and having examined Bell's Theorem and its role in identifying those systems which are nonlocal, we now describe the buildable, deterministic quantum box that is the focus of this thesis: the Q Box.

5.1 The Design of the Q Box

The Q Box is a 3x3x3 master cube made up of 27 smaller cubes, with one extra small cube on the side. The 27 cubes are set within a framework that allows them to be manipulated from all sides; push the extra cube in at any of the 9 possible positions on any of the 6 possible faces, and another cube pops out of the opposite side.

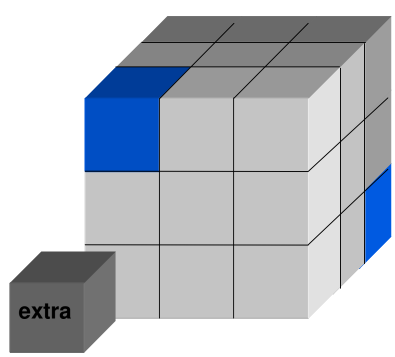


Figure 5-1 The Q Box

Two of the smaller cubes in the Q Box are special; they each carry polarization characteristics. The rest of the smaller cubes that make up the Q Box are identical and unmarked. These unmarked cubes will act as placeholders later as we shuffle the polarized cubes around.

There are four possible polarizations on the special cubes: up, down, left and right. Each of the two special cubes will have one vertical and one horizontal polarization each. Of the $2^4 = 16$ possible combinations of these polarizations on the two cubes, only four of them are correlated such that the two cubes carry opposite polarizations in both the horizontal and vertical directions. We will limit the polarization possibilities to these four special correlated cases in order to match the entangled pair properties we encountered in Mermin's Box.

Here is one possible configuration for the polarized cubes:

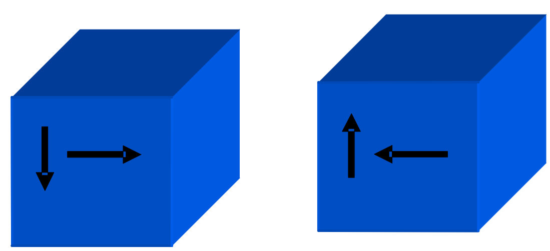


Figure 5-2 Two of the small cubes that make up the Q Box carry hidden variables.

The cube on the left is vertically polarized down (negative) and horizontally polarized to the right (positive). The cube on the right carries polarization properties that are opposite those of its partner. Again, we have no way of knowing where in the Q Box these special polarized cubes are located, nor do we know what their horizontal and vertical values are, ***but we do know that they are strictly correlated with each other.*** If one of the marked cubes carries the attributes of up-right, the other cube is without any doubt down-left. These polarizations are local hidden variables.

Correlation	State 1	State 2	State 3	State 4
Cube #1	↑ →	↓ →	↑ ←	↓ ←
Cube #2	↓ ←	↑ ←	↓ →	↑ →

Figure 5-3 Table of the four possible correlated states for the two special cubes.

This table enumerates the four possible strictly correlated configurations for the two cubes. As explained, these are the only four permutations out of the 16 possible that satisfy our entanglement model. The colors of the arrows characterizing the polarizations correspond to whether they are in the positive (red) direction or the negative (green) direction. As previously stated, up is the positive vertical direction and to the right is the positive horizontal direction. This convention will be used in later calculations.

The two labeled cubes are blindly loaded into the Q Box in random positions. It is important that the labeled cubes are loaded into the Q Box in a way that is unpredictable. It makes the correlation between the two cubes the only information that we have. This is the essence of our model of entanglement. In Fig. 4.1, the two unique cubes are loaded in opposite corners of the Q Box, but the initial positions of the two special, polarized cubes can be anywhere within the Q Box.

5.2 Deterministic Manipulation of the Q Box

Now we begin to manipulate the Q Box. We push the extra, outside cube in at one of the 54 possible entry sites and catch the one that comes out of the opposite side. This becomes the new pusher. Without changing the orientation of the outside cube, (since that could change the vertical-horizontal convention), we push it back into the Q Box at another randomly selected site. The blind loading of the Q Box and shuffling are together the equivalent of taking the entangled pairs to 'opposite sides of the galaxy' as per Mermin's Box. We continue to shuffle the Q Box in this manner until one of the special cubes pops out. At this point, alarms go off and the system stops. The position of the other special cube that is still inside the Q Box is identified, and three Bell's Inequality type measurements are made.

5.3 Measurement of the Q Box

Here is Bell's Inequality again:

$$\left|E_{HV}\left(\hat{a},\hat{b}\right)-E_{HV}\left(\hat{a},\hat{c}\right)\right|\leq 1+E_{HV}\left(\hat{b},\hat{c}\right). \tag{5.1}$$

To use Bell's Inequality as a test for the quantum vs. classical behavior of the Q Box, we need to make three binary measurements: $\hat{a}$, $\hat{b}$ and $\hat{c}$. Then we must calculate the expectation values of the joint measurements $\hat{a}\hat{b}$, $\hat{b}\hat{c}$ and $\hat{a}\hat{c}$, put them into the equation, and see if a violation results.

Our $\hat{a}$ measurement will be the vertical polarization of the ***<u>outside cube</u>***, i.e. the special cube that popped out and started the measurement process. Up (red) will be +1;

while down (green) is -1. The $\hat{b}$ measurement is the horizontal polarization of the ***inside cube***, i.e. the special cube that is still in the Q Box. Both $\hat{a}$ and $\hat{b}$ are local measurements. We assume that making the $\hat{a}$ and $\hat{b}$ measurements wipes that information from the polarized cubes. We will discuss the reasons for this later in the next section.

The third measurement, $\hat{c}$, is the nonclassical element of the Q Box. It is ***nonlocal*** by design. If nonlocality is the quantum ingredient, as we believe, it must act to change the hidden variables such that they can be described by a new probability space; a quantum probability space. In other words, the realization of a nonlocal measurement must change the realization of some of the other measurements of the system. The result of the nonlocal $\hat{c}$ measurement affects the result of the other two joint measurements in the system.

Our nonlocal measurement, $\hat{c}$, is a joint measurement of the horizontal value for the outside cube (cube #1) and the vertical value of the inside cube, (cube #2). These are the values left on each cube after the first two measurements are made and that information subsequently erased. Because we consider both values at once, and the two cubes could be, in principle, far away from each other, this is a nonlocal measurement. But it is a measurement that could be made by some stretch of the imagination as follows:

Assume that the cubes are made of a transparent material, with the polarizations drawn as lines in red and green. Once the first two measurements are made, we are left with only a horizontal polarization on the outside cube and a vertical polarization on the inside cube, as shown below.

Figure 5-4 Only half of the original Information is left on the special cubes after measurement

Now we shine a flashlight through the two cubes. The flashlight has two bulbs, one red and one green. The flashlight selects from the two colors randomly at every measurement. The red or green light passes through both cubes, travels through a quarter-wave plate and then projects onto a screen. We only have access to the projection on the screen. We do not know what color is shining from the flashlight. The quarter-wave plate acts to rotate the arrows 45 degrees, making it so that if you see only one diagonal image, it is impossible to know which of the two cubes produced that image. The three possible images are: a diagonal cross (if both polarizations have the same value, and that value is the opposite color of the flashlight), a diagonal slash (if one polarization is positive and the other negative), or no image at all (if both polarizations are the same value as the flashlight).

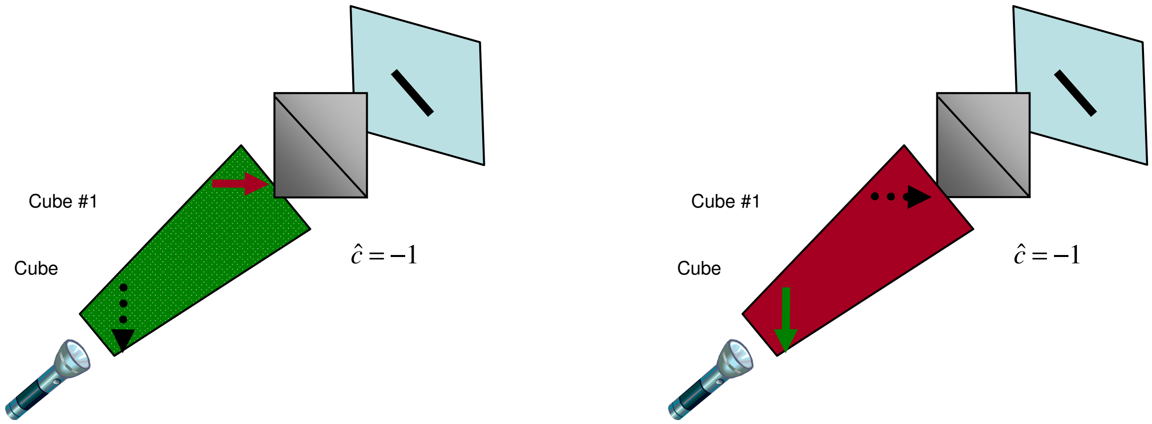


Figure 5-5 Two of four possible nonlocal measurements. If the values of the polarizations do not match, either color of flashlight will reveal ½ of an X shape on the screen. The $\hat{c}$ measurement will be (-1).

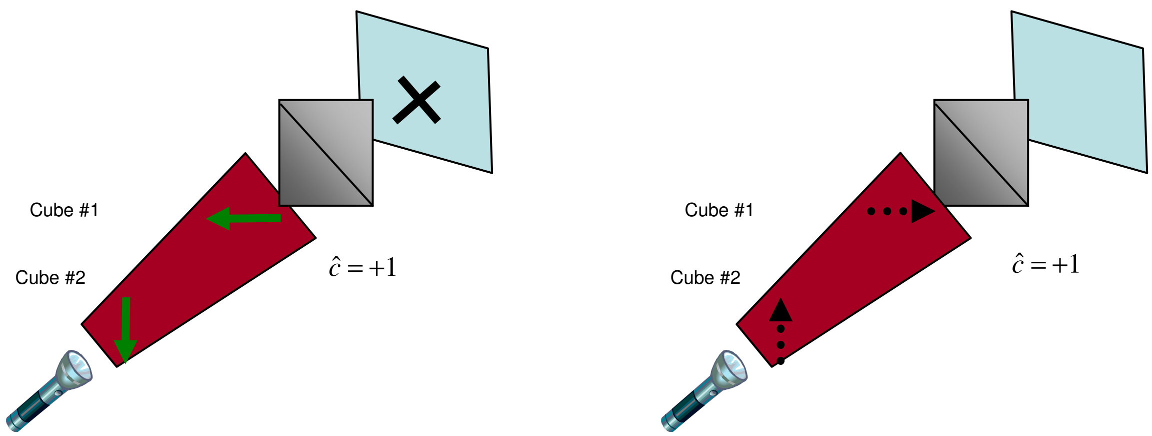


Figure 5-6 Two of four possible nonlocal measurements. If the values of the polarizations match, there will either be an X shape on the screen (for the case where the flashlight color also differs from the polarization values) or nothing at all (for the case where the flashlight color matches the polarization values.

In the case that the cubes are either both positive or both negative, either an X will show up on the screen, or the screen will be blank and one can know that the cubes had matching values. This is a $\hat{c}$ measurement of +1. On the other hand, if only one diagonal slash shows up on the screen, the experimenter knows that the values of the two cubes did not match, and the $\hat{c}$ measurement is -1.

5.4 The Necessity of Measurement Induced Information Loss

That the polarization traits are destroyed upon measurement is actually a subtle, tricky requirement for our deterministic Q Box. In the lab, the scientist does not have the luxury of putting a particle or photon in some kind of stasis box where it can be measured repeatedly over and over. Instead it passes through the measurement apparatus, gives up its information and is either destroyed or changed beyond recognition and continues on its way, with no way of catching that particular particle or photon[10] again. In contrast, our two special cubes each get measured twice; once with an individual measurement and once as part of a joint measurement. If the information does not disappear after measuring $\hat{a}$ and $\hat{b}$ there is too much information left on the cube, and the $\hat{c}$ measurement need not be nonlocal; it can be calculated just by looking at either of the cubes and remembering that the other is correlated opposite to it. We would be back to ordinary, classical correlation instead of entanglement. So the question becomes, "How can we destroy the attributes upon measurement? What deterministic processes might work like this?"

Here are some ideas. We could use a paint that disappears when subjected to light of a certain wavelength, in which case the horizontal and vertical polarizations would need to be inked with paints with different "destruction coefficients" so that the measurement of $\hat{a}$ doesn't destroy the information needed later for $\hat{c}$. Or we could use a low-tech solution like peel-off stickers. Both of these solutions would require a third

[10] What is a photon, anyway?

entity to make the nonlocal $\hat{c}$ measurement, someone that had no knowledge of the value of the previous measurement. Or, since we are just brainstorming here, why not give Alice and bob mind-wipe serums after each $\hat{a}$ and $\hat{b}$ measurement?

Another option would be to use a paint that is only visible when illuminated with a certain frequency of light, say ultraviolet or infrared. Then the vertical polarization could be painted on the cubes with paint sensitive to ultraviolet and the horizontal polarization could be marked with paint that only glows under infrared. The $\hat{a}$ measurer would have an ultraviolet light wand and the $\hat{b}$ measurer an infrared light source. The person who makes the nonlocal measurement would need both.

5.5 The Q Box Rules

To have a quantum probability space that violates Bell's Inequality, the result of our nonlocal measurement must somehow change the outcome of another measurement. Therefore, part of our nonlocal measurement process is the application of three rules, which we elevate in importance by capitalizing them: the Q Box Rules. They are:

1. *If* $\hat{a}$ *and* $\hat{b}$ are being measured jointly, their values remain unchanged.
2. *If* $\hat{a}$ is being measured with $\hat{c}$, its spin value is forced to match that of $\hat{c}$.
3. *If* $\hat{b}$ is being measured with $\hat{c}$, its spin value is forced to be opposite that of $\hat{c}$.

The Q Box Rules are reminiscent of the type of rules encountered in Game of Life simulations: they are simple, they are binary (Q Box results are ± 1 whereas Game of

Life results are alive/dead), they are enforced through a joint measurement and the results of measurement happen simultaneously at the end of each time step.

Following is a tabulation of the results of the three measurements given the four possible states of the two entangled Q Box cubes, both before and after application of The Q Box Rules.

Cube #1 Cube #2	↑ → ↓ ←	↑ ← ↓ →	↓ → ↑ ←	↓ ← ↑ →
$\hat{a}$	+1	+1	-1	-1
$\hat{b}$	-1	+1	-1	+1
$\hat{c}$	-1	+1	+1	-1
$\hat{a}*\hat{b}$	-1	+1	+1	-1
$\hat{b}*\hat{c}$	+1	+1	-1	-1
$\hat{a}*\hat{c}$	-1	+1	-1	+1

Figure 5-7 A Classical System: measurement values *before* application of The Rules.

Before the application of the Q Box Rules, our system is classical, and Figure 5-7 shows what one would expect from a deterministic, local, classical system. The measurements $\hat{a}$, $\hat{b}$ and $\hat{c}$ are independent of each other and the joint measurements reflect this. Recall from our discussion of Bell's Theorem in Chap. 2 that the expectation value of $(\hat{a}\hat{b})$ is the sum of all the joint measurements of $(\hat{a}\hat{b})$ divided by the number of measurements. If we assume that each of the four possible spin configurations are equally likely then the expectation values for $(\hat{a}\hat{b})$, $(\hat{b}\hat{c})$ and $(\hat{a}\hat{c})$ are all identically equal to zero since in each case there are two values of +1 and two values of -1; this is due to the high degree of symmetry in the system. (See Fig. 5.7)

$$\frac{+1+1-1-1}{4}=0. \tag{5.2}$$

When these expectation values are placed into Bell's Inequality, the inequality is satisfied, as indeed it should be for any well-behaved classical system:

$$\begin{gathered}\left|E_{HV}\left(\hat{a},\hat{b}\right)-E_{HV}\left(\hat{a},\hat{c}\right)\right|\leq 1+E_{HV}\left(\hat{b},\hat{c}\right)\\ |0-0|\leq 1+0\\ 0\leq 1\end{gathered} \tag{5.3}$$

Now observe what happens when we apply the Q Box Rules. These rules work under the assumption that the *correlation* is the most fundamentally 'real' attribute of our Q Box, not the specific hidden variables themselves. In other words, the *correlations,* or expectation values, change (guide, command, influence, control) the *realizations* of the hidden variables.

Cube #1 Cube #2	↑ → ↓ ←		↑ ← ↓ →		↓ → ↑ ←		↓ ← ↑ →	
$\hat{a}$	+1	With **b** **+1** With **c** **-1**	+1	With **b** **+1** With **c** **+1**	-1	With **b** **-1** With **c** **+1**	-1	With **b** **-1** With **c** **-1**
$\hat{b}$	-1	With **a** **-1** With **c** **+1**	+1	With **a** **+1** With **c** **-1**	-1	With **a** **-1** With **c** **-1**	+1	With **a** **+1** With **c** **+1**
$\hat{c}$	-1		+1		+1		-1	
$\hat{a}*\hat{b}$	-1		+1		+1		-1	
$\hat{b}*\hat{c}$	-1		-1		-1		-1	
$\hat{a}*\hat{c}$	+1		+1		+1		+1	

Figure 5-8 A Quantum System: measurement values *after* application of the Q Box Rules.

Notice in Figure 5-8 that the hidden variables $\hat{a}$ and $\hat{b}$ change based on their correlation with the nonlocal $\hat{c}$. $\hat{a}$ must match the polarization value of $\hat{c}$ and $\hat{b}$ must be the opposite polarization value from $\hat{c}$, as stated in the Q Box Rules. In all the Rules change only four of the twelve correlation values (shown in red) from those in the classical system in Fig. 5.7, but this is enough to transform this version of the Q Box from classically deterministic to quantum.

At this point we should admit that the deterministic system as advertised in the beginning of this thesis would best be called quasi-deterministic. The Q Box starts out appearing quite normal, classical, well-behaved and deterministic. However, the application of the Rules which change the probability space require part of the *mechanism itself* to be outside of the system. Consider the archetypal deterministic system: clockworks. Once a clock is wound, it ticks away until friction within the system causes it to wind down, each click of the toothed gear leading to the next in a perfectly predictable way. On the other hand, the Q Box clockworks require a measurement and an adjustment at every tick along the way. This will be discussed in greater detail in Chap. 6 and 7.

After application of the Q Box Rules the expectation values become:

$$
\begin{aligned}
E_{HV}\left(\hat{a},\hat{b}\right) &= \frac{-1+1+1-1}{4} = 0 \\
E_{HV}\left(\hat{b},\hat{c}\right) &= \frac{-1-1-1-1}{4} = -1 \\
E_{HV}\left(\hat{a},\hat{c}\right) &= \frac{+1+1+1+1}{4} = +1
\end{aligned}
\tag{5.4}
$$

When evaluated in Bell's Inequality, these new expectation values yield:

$$
\begin{gathered}
\left|E_{HV}\left(\hat{a},\hat{b}\right) - E_{HV}\left(\hat{a},\hat{c}\right)\right| \leq 1 + E_{HV}\left(\hat{b},\hat{c}\right) \\
|0-1| \leq 1 + (-1) \\
1 \leq 0
\end{gathered}
\quad ,
\tag{5.5}
$$

which is a clear violation of Bell's Inequality. Our Q Box is now behaving in a quantum manner. The four changes based on the Q Box Rules were just enough to break the high level of symmetry in our system.

5.6 Q Box Discussion

Our nonlocal measurement and the accompanying Q Box Rules are completely *ad hoc* and at first glance seem rather arbitrary. In the previous section we stated that the *correlations* change the *realizations*. Does this mean our hidden variables were not 'real'? After all, we brainstormed several ways of painting the polarizations onto the cubes in Sec. 5.4. What we once considered to be hard-coded initial conditions for the hidden variable polarizations now appear to be more like initial preferences which are subject to change based on the nonlocal requirements of the system.

Cube #1 Cube #2	↑ → ↓ ←	
$\hat{a}$	+1	With b **+1**
		With c **-1**
$\hat{b}$	-1	With a **-1**
		With c **+1**
$\hat{c}$	-1	
$\hat{a}*\hat{b}$	-1	
$\hat{b}*\hat{c}$	-1	
$\hat{a}*\hat{c}$	+1	

Figure 5-9 The measurements and correlation values for one possible Q Box configuration.

Take for example the configuration state shown in the first column of Figure 5-9, our tabulation from after the Q Box Rules were applied. We have reproduced just the first column in Figure 5-9 above. The vertical polarization of cube #1, ($\hat{a}$) starts out as +1 or 'up'. It remains so if compared or measured jointly with ($\hat{b}$), the other classical entity in the Q Box system. But if one asks a question about $\hat{a}$'s quantum nature by evaluating it jointly with the nonlocal $\hat{c}$, it is forced to flip 'down' to (-1) to match $\hat{c}$.

In other words, in our Q Box the question "What is your polarization state" will always receive the same classical answer if the question is asked of $\hat{a}$ by $\hat{b}$ (as in a joint measurement $\hat{a}\hat{b}$.) On the other hand, if the question is posed by $\hat{c}$ in the joint measurement ($\hat{a}\hat{c}$), then $\hat{a}$ will return its classical value only if matches that of $\hat{c}$, (which it will 50% of the time) as if $\hat{c}$ wields some sort of creepy quantum peer pressure!

This is a little bit reminiscent of the Copenhagen interpretation, in which a state is not determinable until it collapses upon measurement. The two special cubes in our Q Box do have hidden variables attached to them which are predetermined. These can be thought of as "classical preferences". But these inclinations are subject to change upon quantum notice. If all one can see is the end result of the final measurement though, the two systems, Copenhagen Quantum and Q Box, look the same, and the experimenter has no way of knowing whether the particle was originally predisposed one way or the other or was completely undecided until measurement; particles in "a state of superposition," using the quantum vernacular, would manifest themselves at this level in a way that is indistinguishable from our predisposed but easily influenced Q Box particles.

The three hidden variables, $\hat{a}$, the vertical polarization of the outside cube, $\hat{b}$, the horizontal polarization of the inside cube, and $\hat{c}$, our joint nonlocal measurement, with which we chose to measure our Q Box, together comprise only one of several sets of $\hat{a}$, $\hat{b}$ and $\hat{c}$ measurements which we identified during the course of this research, all of which worked effectively together to violate Bell's Inequality. The measurement set of

$\hat{a}$, $\hat{b}$ and $\hat{c}$ used throughout this thesis had the advantage of being the easiest to describe, illustrate and program into our computer simulation . But we would like to stress that possible measurement schemes appear to be limited only by the imagination. We identified measurement schemes that involved Left-Hand-Rule constraints, projections onto the complex plane, odd/even considerations, etc. In every case the measurement sets were a combination of two classical measurements (like $\hat{a}$ and $\hat{b}$) combined with one nonlocal measurement ($\hat{c}$) that acted to break the symmetry of the system and violate the inequality. We are left to wonder what type of measurement scheme Nature might prefer.

5.7 An Important Theorem by Suppes and Zanotti

It appears that nonlocal constraints used to force a quantum probability space like those described in the last section are not wholly unexpected. Patrick Suppes and Mario Zanotti have provided us with an important theorem with an accompanying corollary on hidden variables that is pertinent to this research. Their corollary suggests that not only is a nonlocal measurement/connection/control mechanism to be expected in quantum systems with hidden variables, it is crucial.

In their paper "When are Probabilistic Explanations Possible?" [27] Suppes and Zanotti discuss causal variables and the necessity that such variables yield conditionally independent data. Here is what Suppes and Zanotti have to say about hidden variables in quantum mechanics and how Bell's Inequality works as a "necessary but not sufficient condition for conditional independence."

> *. . . [T]he search for common causes in quantum mechanics is the search for hidden variables. A hidden variable that satisfies the common*

cause criterion provides a satisfactory explanation "in classical terms" of the quantum phenomenon. Much of the earlier discussion of hidden variables in quantum mechanics has centered around the search for deterministic underlying processes, but for some time now the literature has also been concerned with the existence of probabilistic hidden variables. It is a striking and important fact that even probabilistic hidden variables do not always exist when certain intuitive criteria are imposed. One of the simplest examples was given by Bell in 1971, who extended his earlier deterministic work to construct an inequality that is a consequence of assuming that two pairs of values of experimental settings in spin-1/2 experiments must violate a necessary consequence of the common cause criterion, that is, the requirement that a hidden variable render the data conditionally independent. It is easy to show that Bell's inequality is a necessary but not sufficient condition for conditional independence.

The implication of the theorems and corollary derived by Suppes and Zanotti is that there exists some irreducible nonlocal connection between the variables. Because quantum mechanical systems that violate Bell's Inequality also precisely violate the conditions of the Suppes and Zanotti theorems, the results of this thesis can provide insight into details of the probability structure of measurements on quantum mechanical systems.

Now, to show that our nonlocal measurement protocol works in a general way, we would like to revisit Mermin's Box. We will demonstrate that, using BI2 and another measurement scheme that combines classical and nonlocal elements with the application of two rules reminiscent of the measurement protocol we used on the Q Box, we can obtain Mermin's quantum statistics from his beginning deterministic hidden variable system.

Chapter 6

Mermin's Box Revisited

In Chap.3 of this thesis, we described Mermin's Box as a pedagogical structure intended to showcase the impossibility of obtaining quantum results from a classical system with hidden variables. In Chap. 5 we described how our classical Q Box with hidden variables becomes a quantum system by applying nonlocal rules. In this chapter we would like to pose a question. Can the same technique be used on Mermin's Box? Is there a nonlocal measurement and rule combination that produces the unexpected quantum results of Mermin's Box? Will the technique work even if we start with the deterministic system Mermin implies is inside his box in the beginning, instead of the spin-1/2 particles he unveils later?

Recall that in Chap. 2 we investigated two different formulations of Bell's Theorem. In Chap. 5, we used the original inequality proposed by Bell,

$$\left|E_{HV}\left(\hat{a},\hat{b}\right)-E_{HV}\left(\hat{a},\hat{c}\right)\right|\leq 1+E_{HV}\left(\hat{b},\hat{c}\right) \tag{6.1}$$

as a means to evaluate the quantum character of the Q Box by using three measurements: the classical measurements $\hat{a}$ and $\hat{b}$ and the nonlocal measurement $\hat{c}$. To do this we needed three measurement devices: a vertical analyzer to measure $\hat{a}$, a horizontal analyzer to measure $\hat{b}$ and the nonlocal flashlight used to make the joint measurement $\hat{c}$. The value of the nonlocal measurement $\hat{c}$ acted to change the values of the classical $\hat{a}$ and $\hat{b}$ measurements according to these three rules:

1) *If* $\hat{a}$ *and* $\hat{b}$ are being measured jointly together, their values remain unchanged.
2) *If* $\hat{a}$ is being measured with $\hat{c}$, its spin value is forced to match that of $\hat{c}$.
3) *If* $\hat{b}$ is being measured with $\hat{c}$, its spin value is forced to be opposite that of $\hat{c}$.

Cube #1 Cube #2	↑ → ↓ ←		↑ ← ↓ →		↓ → ↑ ←		↓ ← ↑ →	
$\hat{a}$	+1	With **b** **+1**	+1	With **b** **+1**	-1	With **b** **-1**	-1	With **b** **-1**
		With **c** **-1**		With **c** **+1**		With **c** **+1**		With **c** **-1**
$\hat{b}$	-1	With **a** **-1**	+1	With **a** **+1**	-1	With **a** **-1**	+1	With **a** **+1**
		With **c** **+1**		With **c** **-1**		With **c** **-1**		With **c** **+1**
$\hat{c}$	-1		+1		+1		-1	
$\hat{a}*\hat{b}$	-1		+1		+1		-1	
$\hat{b}*\hat{c}$	-1		-1		-1		-1	
$\hat{a}*\hat{c}$	+1		+1		+1		+1	

Figure 6-1 Table showing how the $\hat{c}$ measurement controls the realization of $\hat{a}$ and $\hat{b}$.

For the nonlocal measurement of the Q Box, we were able to identify a joint measurement over two previously unused hidden variables; the horizontal polarization of cube #1 and the vertical polarization of cube #2. In other words, we made three measurements over four hidden variables. Bell's original Inequality, with its three measurement possibilities was just what we needed to evaluate our Q Box.

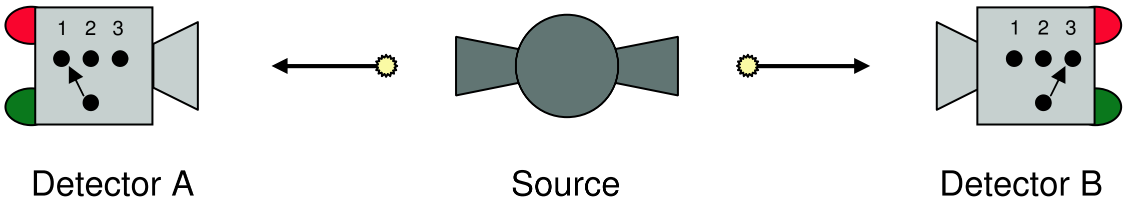


Figure 6-2 Mermin's Box

In the case of Mermin's Box, though, the situation is different. Mermin's Box contains two detectors, and each detector has three possible settings. In Chap. 2 we discussed the fact that the particles emitted from the two sources in Mermin's Gedankenexperiment are assumed to each carry matching three-bit sets of information. The existence of these twin three-bit information sets was inferred from the fact that there are three possible settings on the two detectors, and the particles trigger matching flashes 100% of the time when the detectors are set the same. Since these hidden variable sets are assumed to be equivalent, there is no way to make a meaningful nonlocal joint measurement over both of them; any attempt to compare the 'leftover' hidden variables as we did in the Q Box will always yield trivial equivalent answers and no new information.

So, we need a new nonlocal measurement scheme and some version of Bell's Inequality that utilizes the hidden variables available to us in the system of Mermin's Box. Instead of taking *three* measurements over *four* hidden variables, as in our Q Box, for our Mermin's Box measurement scheme we will take *four* measurements over *three* hidden variables; two measurements from each detector. We'll call the measurements from detector #1 $\hat{a}$ and $\hat{a}'$, and those measurements taken on detector #2, $\hat{b}$ and $\hat{b}'$. As for which version of Bell's Inequality to use, we have already discussed in Chap. 3 Bell's second Inequality, or BI2:

$$\left|E_{HV}\left(\hat{a},\hat{b}\right)-E_{HV}\left(\hat{a},\hat{b}'\right)\right|+\left|E_{HV}\left(\hat{a}',\hat{b}'\right)+E_{HV}\left(\hat{a}',\hat{b}\right)\right|\leq 2. \tag{6.2}$$

BI2 is a comparison of the correlations between four hidden variables: $\hat{a}$, $\hat{a}'$, $\hat{b}$ and $\hat{b}'$, and meets our requirements.

The nonlocal measurement, though, will take a little more thought. As discussed previously, the hidden variable sets are equivalent. Making meaningful nonlocal joint measurements over pairs of them is impossible. Therefore, we are forced to incorporate our measurement devices in our nonlocal measurement scheme.

Is this acceptable? In the realm of quantum science, yes, it is acceptable. The seemingly necessary inclusion of measurement devices in the quantum state function is part of what makes quantum mechanics so unsettling for many scientists. In Griffiths' *Introduction to Quantum Mechanics*, he says:

> *The measurement process plays a mischievous role in quantum mechanics: It is here that indeterminacy, nonlocality, the collapse of the wave function, and all the attendant conceptual difficulties arise. Absent measurement, the wave function evolves in a leisurely and deterministic way, according to the Schrodinger equation and quantum mechanics looks like a rather ordinary field theory. . . It is the bizarre role of the measurement process that gives quantum mechanics its extraordinary richness and subtlety."* [28]

Another favorite quote is from *The Quantum Challenge*,

> *"In an evocative phrase, Bohm has described a quantum state as a "set of potentialities." Following up on this point of view, Shimony and others advocate thinking of measurements as* *actualizing one of these potentialities."* [20]

"Actualizing a potentiality" is precisely what we did with our nonlocal flashlight measurement in the Q Box, as shown in Fig. 6.1. We will do the same with Mermin's Box. We will make a measurement which will *actualize a potentiality*. But, in the Mermin's Box measurement scheme the rules will include the measurement device in an even more fundamental way than in the Q Box.

Det#1	Det#2	RRR	RRG	RGR	RGG	GRR	GRG	GGR	GGG
1	1	RR	RR	RR	RR	GG	GG	GG	GG
1	2	RR	RR	RG	RG	GR	GR	GG	GG
1	3	RR	RG	RR	RG	GR	GG	GR	GG
2	1	RR	RR	GR	GR	RG	GR	GG	GG
2	2	RR	RR	GG	GG	RR	RR	GG	GG
2	3	RR	RG	GR	GG	RR	RG	GR	GG
3	1	RR	GR	RR	GR	RG	GG	RG	GG
3	2	RR	GR	RG	GG	RR	GR	RG	GG
3	3	RR	GG	GG	GG	RR	GG	RR	GG

Figure 6-3 The measurement results of Mermin's Box assuming a strictly local mechanism.

Here again is the table showing all of the possible detector settings, hidden variable combinations and the results given a strictly deterministic mechanism inside. In

Chap. 2 we discussed how such a deterministic mechanism can never produce the observed output of the box:

> ***Observation 1****:* *Every time the detectors are identically set they flash the same color, although* ***which*** *color is impossible to predict.*
>
> ***Observation 2****:* *When the detectors are turned to different settings, they flash the same color about 25% of the time.*

As can be seen in Fig. 6.3 above, and as was addressed in Chap. 2, the realization of Observation 1 requires matched 3-bit hidden variable sets. The same 3-bit sets requisite for producing these results conflict with Observation 2; instead of generating same colored flashes when the detectors are turned to different settings the required 25% of the time, they yield matching colors 50% of the time. As with the Q Box, we must find a nonlocal measurement that will affect the realizations of these hidden variables; measurement must *actualize the potentialities*.

Here is the background for our Mermin's Box Rules:

Given that Red is 'odd' and Green is 'even', each of the three bits of information carried by the particles is 'odd' or 'even'. These are local variables, since in principle they are carried along with the particles. In the same way, the results of measurement, the flashing colored signal from the detectors, can also be considered odd or even. If one detector flashes red and the other green, that is an odd data event. If both detectors flash either red or green, that is an even data event, since odd + odd always is even and even + even is also always even.

For our nonlocal measurement, we will take the sum of the values, even or odd, of the two detectors. If detector #1 is set at one and detector #2 is set at three, the sum of the two, 'odd' + 'odd' supplies a *nonlocal* measurement of 'even'. Here is a new table reflecting these conventions:

Det#1	Det#2	RRR	RRG	RGR	RGG	GRR	GRG	GGR	GGG
1 even	1	RR	RR	RR	RR	GG	GG	GG	GG
1 odd	2	RR	RR	RG	RG	GR	GR	GG	GG
1 even	3	RR	RG	RR	RG	GR	GG	GR	GG
2 odd	1	RR	RR	GR	GR	RG	GR	GG	GG
2 even	2	RR	RR	GG	GG	RR	RR	GG	GG
2 odd	3	RR	RG	GR	GG	RR	RG	GR	GG
3 even	1	RR	GR	RR	GR	RG	GG	RG	GG
3 odd	2	RR	GR	RG	GG	RR	GR	RG	GG
3 even	3	RR	GG	GG	GG	RR	GG	RR	GG

Figure 6-4: Even and odd conventions for Mermin's Box.

Following are the Mermin's Box Rules that act to change the realizations of the hidden variables. They are in the form of one rule and one exception:

Rule: ***If the sum of the joint detector settings is odd, the sum of the two detector flashes must also be odd.***

Exception: ***If all three hidden variables are even, the sum of the two detector flashes will also be even, no matter what the detectors are set at.***

After application of the rules, the results will be those shown in the table below. The figures in blue denote those hidden variables that change based on the nonlocal measurement across the two detectors. Now the data matches the original specifications of Mermin's Box; i.e., when the detectors are set the same, the flashes match. When the detectors are set differently, the detectors flash the same color only 25% of the time.

Det#1	Det#2	RRR	RRG	RGR	RGG	GRR	GRG	GGR	GGG
1 even	1	RR	RR	RR	RR	GG	GG	GG	GG
1 odd	2	RR/RG	RR/RG	RG	RG	GR	GR	GG/RG	GG
1 even	3	RR	RG	RR	RG	GR	GG	GR	GG
2 odd	1	RR/RG	RR/RG	GR	GR	RG	GR	GG/RG	GG
2 even	2	RR	RR	GG	GG	RR	RR	GG	GG
2 odd	3	RR/RG	RG	GR	GG/RG	RR/RG	RG	GR	GG
3 even	1	RR	GR	RR	GR	RG	GG	RG	GG
3 odd	2	RR/RG	GR	RG	GG/RG	RR/RG	GR	RG	GG
3 even	3	RR	GG	GG	GG	RR	GG	RR	GG

Figure 6-5 Mermin's Box data after application of the rules.

At the beginning of this chapter we posed some questions: "Can the same technique used on the Q Box to make it behave in a nonclassical manner be used on Mermin's Box? Is there a nonlocal measurement and rule combination that produces the unexpected quantum results of Mermin's Box, even if we start with the deterministic system Mermin implies is inside his box in the beginning, instead of the spin-1/2 particles he unveils later?"

Again, as is the case with the Q Box, the answer appears to be, "Yes! If . . ." ***If*** a nonlocal measurement can be identified that produces the right statistics and can serve as some kind of trigger or sorter or enforcer of the hidden variables. Just how that mechanism might actually work is unclear. In our reworking of Mermin's Box, it is as if there is some sort of implicit Affirmative Action Program for the Advancement of Oddness going on inside the box. Only the triplet of Even-Even-Even is immune. It should be noted that this is the only one of the possible particle configurations which doesn't carry an odd bit within itself.

For these changes to happen deterministically, the particles encountering a joint odd setting must either shuffle the order of their variables *in a coordinated way* so that the correct (one odd, one even) values can be 'scanned' by each detector, or the hidden variables must be able to change so that the particles jointly comply with this odd company culture. Either scenario implies communication and complicity between the two sets of hidden variables and/or the detectors.

This leads to a second obvious question. At what level in the system is the oddness enforced? Do the particles themselves change their hidden variables, or at least decide which of the two options to present? This would imply a nonlocal knowledge of what is happening with their twin. It seems more likely than having the detector be in charge of the 'presentation of the colors'.

Could there an omniscient Maxwell's Demon floating above the experimental Mermin's Box setup? Such a Demon would need to be able to discern both the detector settings and the states of the particles and then direct affairs like an air traffic controller with a penchant for oddness.

Is it possible that there is a preferred state that the particles like to be in? That given the same three colored faces (hidden variables) to choose from to present to the world, the particle pairs prefer to be individualistic, like self-conscious identical twins who insist on dressing differently whenever possible.

Or could it be that an odd pair state is somehow more advantageous? From the table above we can draw an energy analogy. It could be construed that it is 'easier' to be odd than even, as if being odd is a lower energy state while remaining even requires more work.

We must finally draw attention to the most glaring cheat of both the Q Box and the reworked Mermin's Box. In both cases we advertised a deterministic system. Hidden variables are certainly in keeping with a deterministic model. Nonlocality, however, is a quantum beast, and we have used it shamelessly throughout. *Nonlocality is the key*. It is like a quantum spring in our otherwise clockwork boxes. Nonlocality is like a sprinkling of quantum pixie dust that allows an otherwise pedestrian deterministic system to completely change its nature.

These two dual systems, the Q Box and Mermin's Box, each contain deterministic components which are influenced in some way by nonlocal effects or interactions. Combining the quantum and deterministic this way has given us insights into the possible interplay between these two realms of nature. These insights will be addressed in our concluding chapter.

Chapter 7

Reality Checks

7.1 Nonlocal Measurement

In this chapter we would like to address all of the criticisms of our model that we can possibly think of. The first candidate for examination is our nonlocal measurement. Is it perhaps cheating to put the nonlocality of our system explicitly into the measurement process? At first glance, Bell's Theorem seems to imply that three measurements are made on isolated particles that are members of an entangled pair. The typical Alice-Bob Bell's Theorem illustrations in popularizing physics books (and in Chap. 2 of this thesis) also make it seem that the scientists 'catch' and 'hold' their half of the particle pair, like some kind of squirmy two-year-old, and that the particle holds patiently still while the scientists subjects it to the three different measurement procedures.

Reality in the lab is a little different. Entangled particles are typically created with either a nonlinear crystal in a down-conversion scheme or with magnets in a Stern-Gerlach approach. Entangled particles are not just pulled out in perfect matching sets from the magic pockets of Alice and Bob. Instead, the arrivals of entangled particles to each scientist are signaled with elaborate timing devices. Experimental noise, dark counts and other experimental vagaries are expected. This makes the 'measurement' process an imprecise game of looking for deviations from a statistical norm. Alice and Bob, who instead of being on opposite sides of the universe are in reality at opposite ends of an optics table, play a much more sophisticated game of 'catch' than the simplistic Alice and Bob illustration implies. One does not 'catch' a photon, one 'bins' detection events. Alice and Bob are not each measuring one of two squirming identical twin two-year-olds; they are in truth each sampling from a stream of multitudes of fidgeting twins. The only sure thing is that every one of the twins in Alice's racing stream has a double somewhere in the torrent passing by Bob, so the experiments become more a statistical study of 'twinness' than a measurement of the particular attributes of any one particular pair of twins.[11]

John Bell [16] understood this, and designed his theorem accordingly. He said,

> *"Unfortunately it is not at present (1976) possible to approach the conditions of the ideal critical experiment. Real counters, real polarization analyzers, and real geometrical arrangements, are together so inefficient that the quantum mechanical correlations are greatly diluted. The counters seldom say 'yes, yes', usually say 'no, no', and say 'yes, no' with a frequency only weakly dependent on the polarizer settings. In these conditions [instead of* $P(a,b)=\pm 1$ *we have]*
>
> $$P(a,b)=1-(\delta(a,b))^2$$
>
> *where* δ *is small and weakly dependent on the arguments,* (a,b)*. The inequality*
>
> $$\left|E_{HV}\left(\hat{a},\hat{b}\right)-E_{HV}\left(\hat{a},\hat{b}'\right)\right|+\left|E_{HV}\left(\hat{a}',\hat{b}'\right)+E_{HV}\left(\hat{a}',\hat{b}\right)\right|\le 2.$$
>
> *is then trivially satisfied. So it is only by allowing (in effect) for various inefficiencies in conventional ways, and so extrapolating from the real results to hypothetical ideal results, that the various experiments can be said to 'test' the inequality. But the results are nevertheless of great interest[12]. Compensating failures could be imagined, of the conventional quantum mechanics of spin correlations and of the conventional*

[11] "If your experiment needs statistics, then you ought to have done a better experiment." *Ernest Rutherford (1st Baron Rutherford of Nelson)* [29]

[12] Bell: "Several of the real experiments are of great elegance. To hear of them (not in schematic terms from a theorist but in real terms from their authors) is, to borrow a phrase from Professor Gilberto Bernardini, a spiritual experience." [16]

phenomenology of the instruments, which would make the practical experiments irrelevant. But that would seem an extraordinary conspiracy.

Bell also said, while expressing his discontent with the word 'measurement'[30]:

"When I say that the word 'measurement' is worse than the others. . . I do have in mind its use in the fundamental interpretive rules of quantum mechanics. . ." The first charge against 'measurement', in the fundamental axioms of quantum mechanics, is that it anchors there the shifty split of the world into 'system'; and 'apparatus'.

A second charge is that the word comes loaded with meaning from everyday life, meaning which is entirely inappropriate in the quantum context. When it is said that something is 'measured' it is difficult not to think of the result as referring to some preexisting property of the object in question. This is to disregard Bohr's insistence that in quantum phenomena the apparatus as well as the system is essentially involved."

And this [21]:

"A fine moral concerns terminology. . . I suspect that [people are] misled by the pernicious misuse of the word 'measurement' in contemporary [quantum] theory. This word very strongly suggests the ascertaining of some preexisting property of some thing, any instrument involved playing a purely passive role. Quantum experiments are just not like that, as we learned especially from Bohr. The results have to be regarded as the joint product of 'system' and 'apparatus,' the complete experimental set-up. But the misuse of the word 'measurement' makes it easy to forget this and then to expect that the 'results of measurements' should obey some simple logic in which the apparatus is not mentioned. . . I am convinced that the word 'measurement' has now been so abused that the field would be significantly advanced by banning its use altogether, in favor for example of the word 'experiment.'

Ian Percival, speaking at the International Erwin Schrödinger Institute in Vienna at their 2000 Conference commemorating John Bell, makes it very clear that Bell understood the intrinsic role measurement must play in quantum systems and designed his inequalities accordingly. [31]

"There is a profound distinction between experiments to test weak [abiding by relativistic limits] nonlocality by the violation of Bell inequalities and most other experiments with quantum systems.

Consider for example an experiment to determine the spectrum of an atom. . . The aim of [this] experiment is to determine the properties of [this] quantum system. The classical apparatus used to prepare the *system and to make the necessary measurements is essential, but secondary to obtaining these properties.*

In Bell experiments the converse is true. The aim is to test for violation of the inequalities, which are derived from the (statistical) properties of classical events. . . . The probabilities of the output, given the inputs, are what appear in the Bell inequalities, and it is the location of these events in spacetime that determine the locality or nonlocality.

These classical events are connected by an ancillary quantum system, whose function is to produce their unusual statistical properties. The quantum properties, like the entanglement of particles, or the polarization of photons, or the spins of atoms, are essential, but secondary. The primary result is the violation of an inequality. ***The apparatus as well as the quantum system is essentially involved****.* [Emphasis added.]

This distinction has implications for the analysis of Bell experiments. Real Bell experiments are designed to approximate ideal experiments. But the classical events in a real experiment are usually different from those in the ideal experiment which it simulates. . . For example, in an ideal experiment, it is usually assumed that the detectors detect every particle, but in real experiments they don't.

It follows that the inequalities of the ideal experiment do not always apply directly to the real experiment, and further assumptions are needed to demonstrate weak nonlocality.

[One] way is to recognize that every real experiment that simulates an ideal Bell experiment has its own critical inequalities, [sic] that apply directly to the probabilities for all the outputs of the real experiment. . .

If any one of these inequalities is violated, weak nonlocality has been demonstrated, and no further assumptions are needed."

John Bell died unexpectedly of a cerebral hemorrhage at the age of 62. [32] But, based on his own words and statements like those above, it seems clear that Bell designed his theorem for just the type of model we are proposing.

7.2 Superluminal Communication

Our research implies some form of superluminal communication between the entangled system particles. In our case, this nonlocal communication is between the Q

Box cubes. In the case of Mermin's Box, the communication must either be between the particles or between the detectors or some combination of the two.

Although "spooky-action-at-a-distance" is a socially acceptable topic of polite conversation between physicists, there are varying degrees of belief. Opinions range from 'not relevant to my research' to 'Nonlocality: The God Phone'.[13] Probably the most prevalent viewpoint is that yes, indeed, there seems to be some sort of inexplicable "spooky-action-at-a-distance", but not to worry, as long as information about that action cannot be passed faster than the speed of light, relativity still reigns. We have based this research on the assumption that nonlocality is a fact of nature. But what is the mechanism? We would like to revisit our two analogies from Chap. 2.

Remember our reincarnation of Maxwell's demon as a gym teacher? We will need to give our demonic gym teacher two impish teacher's aides. Let's call them Woodscrew and Tapeworm. Each is standing at one of the detectors with their clipboards and whistles, forcing some of the particles to change their colors if they don't meet their high quantum standards. Of course, to do this effectively, the teacher's aides would need to be in communication with each other in order to know what is happening at the other's detector. Their conversation might sound something like two dockworkers sorting cargo:

"Hey, Woodscrew! I see a red particle approaching. What do you have?"

"Incoming blue. They don't match, so we have a negative--

"How are we doing on our stats? Are we meeting our quantum quota?"

"Nope. This one's gotta change."

Then Woodscrew and Tapeworm would need to make a decision together about which of the incoming particles must change color and then they must somehow enforce or expedite or facilitate this change. Because of the possibility of one particle registering

[13] Try googling "superluminal action at a distance" and see what we mean.

while the other particle is still in transit, it seems most probable given this scenario that the particles themselves carry a duality of color choices with them. This hearkens back to our discussion of the Q Box, where the particles can be construed to have a preferential state that is subject to change if the quantum need arises.

In the space portal docking analogy, some internal computer system would be needed to check the measured color against the color from the other port and then decide whether to report the 'old reality' (the color the particle started out with) or the 'new reality' (the color it must be changed to in order to be statistically quantum). Perhaps the space port comes equipped with a paint shop to take care of ships that are no longer the quantum acceptable color.

Both examples are silly and nonsensical, but serve to illustrate the sticking point of nonlocality. Where/who/what is the mechanism? "When is the mechanism?" is also a valid question. It appears that the communication is superluminal, but is the system weakly nonlocal or nonlocal in a way that implies relativity can be ignored?

We have no answers to these questions. We can only say that it *in our model* we can reproduce quantum effects if we assume that there is some overarching nonlocal connection that somehow controls the realizations of the hidden variables in our model. Whether Nature herself works this way is impossible to say.

In Chap. 8 we will continue our discussion of nonlocal connections as illustrated in the computer simulation of our Q Box.

Chapter 8

A MATLAB Computer Simulation of the Q Box

There is some irony in having the computer simulation discussion so close to the end of this thesis. The computer programming really came first, and played a crucial role in catalyzing the research process by forcing us to think about the possible mechanisms of quantum mechanics as we struggled to model such a system.

When we first began the research process we were endeavoring to duplicate in a model what we perceived to be the most fundamental quantum elements of a spin-1/2 system: entanglement, indistinguishability, and quantum stochasticity. We originally set out to study the effects of marginalization on a completely deterministic, three dimensional system and for this purpose designed an early progenitor of the Q Box.
At the outset, we were more concerned with what it means to be entangled and indistinguishable than we were about the nonlocal aspects of quantum systems. Hidden variables, which have since evolved to become the focus of our research, were of interest in the beginning only as the properties that are entangled. Because of this focus on entanglement, we went through great lengths to write computer code that we hoped would perform the same role that quantum erasure techniques fulfill in quantum optics labs, and some of this is still remnant in the code.

As the research progressed, and we became convinced that the more interesting questions lie in the interplay between hidden variables and nonlocality, our computer model became a springboard back to a more sophisticated Q Box; one in which the hidden variables play a much more important role than simply being the-property-that-is-entangled. At the same time, we have come to view entanglement as merely the manifestation of hidden variables that are being somehow controlled through nonlocal measurements. Entanglement has lost its mystique.

The computer model is written in MATLAB code. Of the many different generations of the code, we have decided that the two simulations that have been most influential to our research are the versions KEEPER and SIMPLEX. Both are included in the appendices. KEEPER is a general model for the Q Box as described in Chap. 5. The second program, SIMPLEX, is a modification that permits weightings of the different possible polarization configurations, allowing us to investigate more complicated probability spaces for the Q Box. After describing the general program, KEEPER, we will discuss the simplex that governed our probability choices for the SIMPLEX program. Finally, we will discuss the modifications made for SIMPLEX.

8.1 The MATLAB Program KEEPER

KEEPER starts with *two* 3x3x3 cubes: redcube and greencube, which are programmed as three variable matrices; redcube(x,y,z) and greencube(x,y,z). The red matrix tracks the vertical polarization characteristics and the green matrix tracks the horizontal polarization characteristics of the two polarized cubes in the Q Box. Recall that in our original Q Box there is also one cube that begins outside and is the first cube to be pushed in. Therefore our program also has one 'outside' cube for both redcube and greencube: redout and greenout. Since both redcube and greencube are controlled by the same shuffling algorithm, this two matrix system is the equivalent of our Q Box with its two polarized cubes, each with one random vertical and one random horizontal component.

We begin by loading the numbers one through twenty-seven into the twenty-seven minicubes that make up redcube and greencube. The position (x,y,z) matrices then look like this:

(1, 1, 1) = 1	(1, 2, 1) = 10	(1, 3, 1) = 19
(1, 1, 2) = 2	(1, 2, 2) = 11	(1, 3, 2) = 20
(1, 1, 3) = 3	(1, 2, 3) = 12	(1, 3, 3) = 21
(2, 1, 1) = 4	(2, 2, 1) = 13	(2, 3, 1) = 22
(2, 1, 2) = 5	(2, 2, 2) = 14	(2, 3, 2) = 23
(2, 1, 3) = 6	(2, 2, 3) = 15	(2, 3, 3) = 24
(3, 1, 1) = 7	(3, 2, 1) = 16	(3, 3, 1) = 25
(3, 1, 2) = 8	(3, 2, 2) = 17	(3, 3, 2) = 26
(3, 1, 3) = 9	(3, 2, 3) = 18	(3, 3, 3) = 27

Figure 8-1 Position matrices and their values for both KEEPER and SIMPLEX.

Using a random generator and a rounding function, redcube's two corner minicubes—those in positions (1, 1, 1) and (3, 3, 3)— are *blindly* loaded with values of 0.1 and 0.2, represent spin-up and spin-down, respectively. A random number generator decides which one goes in each corner after which they are both immediately rounded down to zero. The greencube likewise is randomly assigned two values for the corner minicubes that are less than one; (1, 1, 1) is either 0.3 (spin-right) or 0.4 (spin-left), leaving (3, 3, 3) to be the opposite value. Again, a FLOOR command whitewashes the values into nondescript zeros. This blind load duplicates the beginning of our Q Box, wherein we start with corner cubes each having one unknown vertical (redcube) polarization and one unknown horizontal (greencube) polarization. This gives us a model twice removed from the spin-½ particles our Q Box is loosely based on, with an entangled redcube and an entangled green cube created by 'rounding down' the four special cubes such that the values for them are equally zero and indistinguishable. This becomes our mechanism for 'entanglement'.

Note that although the program is written to follow the description given of the Q Box in that the beginning position of the special cubes is always the two opposing corners (1, 1, 1) and (3, 3, 3), it would be a simple modification to load the randomly up/down, right/left cubes into any two randomly chosen positions in greencube and redcube. We found the one level of randomization to be sufficient.

Once the two special cubes with values less than one are loaded into the corners of redcube and greencube, a 4-digit series is randomly generated to decide where the extra block will enter the master green and red entangled cubes. There are 54 positions on the face of the cubes that can be "pushed"; six faces per cube with nine possibilities on each face. Only the (2, 2, 2) position is invalid as a possible entrance point, since (2, 2, 2) denotes the coordinates for the center of the cube and is therefore inaccessible from the outside. The first number in the 4-digit series is between one and six and designates the direction of entry or face of the cube. The next three numbers are all between one and three and control the x, y, and z coordinates on that face.

One of the three x, y or z coordinates is superfluous. If the first number chosen by the computer is one, then the entry face is the top of the cube; we will be pushing in the –z direction.   In this case only the x and y coordinates are necessary to choose which of the nine possible positions on the cube top will be the pushpoint.

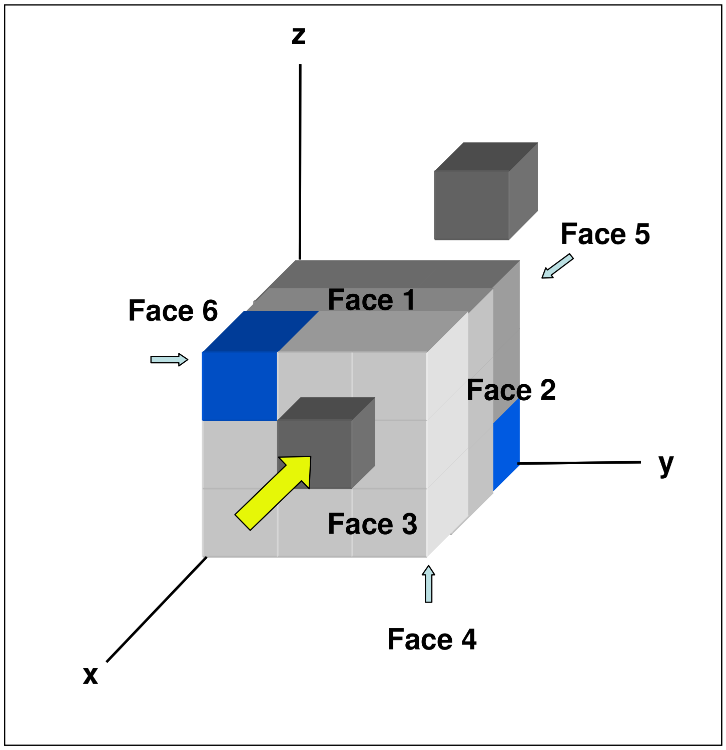


Figure 8-2 The result of the extra cube being pushed into the master cube at position (3, 3, 2, 2).

Figure 8-2 shows the results of the random series (3, 3, 2, 2). The extra cube is being pushed in the -x direction, entering Face 3 at the center position (x=3, y=2, z=2). This forces another cube out from the back of the cube from position (x=1, y=2, z=2). Another random generation picks a new entry point and the shuffling continues.   If at any time the computer calls the forbidden (x=2, y=2, z=2) position as a push point, the series is discarded and another entry point is chosen by the computer.

The shuffling continues, over and over. Each time one of the cubes pops out, it is checked to see if the value is zero. If so, a search mechanism finds the position in the matrix of the other zero valued cube and the two cubes, outside and inside are measured using the same protocol as the Q Box; first the vertical component of the outside cube, which is our $\hat{a}$ measurement, then the horizontal component of the inside cube, which is our $\hat{b}$ measurement, and finally the $\hat{c}$ joint measurement across the other two variables. Expectation values of the joint measurements $\hat{a}\hat{b}$, $\hat{b}\hat{c}$ and $\hat{a}\hat{c}$ are then calculated and the same three rules applied that we used in the Q Box. The results are inserted into Bell's Inequality and the solution checked to see if the inequality is violated or satisfied. Samples of various runs are included in the appendices.

Once the basic model was completed we played with many different variations of the program, each one with differing levels of randomization. For example, in one version of the program we shuffled the cube *n* number of times and then took the next zero out of the box for measurement. In another we measured only every fifth, or fiftieth or five-hundredth zero and observed the trends.

One of the most instructive versions of the program is called SIMPLEX. Before describing the program itself it would be well to consider the Q Box simplex upon which it is based.

8.2 The Simplex

A simplex is a graphical interpretation of the probability space of a system. It shows the effects of weighting different possible outcomes. Our Q Box, as discussed in Chap. 5, is designed such that each of the four possible polarization configurations of the two cubes is equally likely. A simplex shows the effects of having one configuration more likely than the others. These probabilites are called P_1, P_2, P_3 and P_4.

Probabilities	P_1	P_2	P_3	P_4
Cube #1 Cube #2	↑ → ↓ ←	↑ ← ↓ →	↓ → ↑ ←	↓ ← ↑ →
$\hat{a}$	+1	+1	-1	-1
$\hat{b}$	-1	+1	-1	+1
$\hat{c}$	-1	+1	+1	-1
$\hat{a}*\hat{b}$	+1	+1	+1	-1
$\hat{b}*\hat{c}$	+1	+1	-1	-1
$\hat{a}*\hat{c}$	-1	+1	-1	-1

Figure 8-3 Data from nonlocal measurements to be used to compose the simplex.

Above is a table that illustrates with the four columns of red and green arrows the four combinations of polarizations possible in our Q Box, along with the results after our nonlocal measurement and the application of our rules. We will solve the expectation values for the general probabilities P_1, P_2, P_3 and P_4 of each possible configuration. Extracting the information for the expectation values from our table and assuming that $P_4 = 1 - P_1 - P_2 - P_3$, we find the following correlations:

$$\begin{aligned} E(AB) &= 2P_1 + 2P_2 + 2P_3 - 1 \\ E(AC) &= 2P_2 - 1 \\ E(BC) &= 1 + 2P_1 + 2P_2. \end{aligned} \tag{8.1}$$

Inserting these values into Bell's Theorem, $\left|E_{HV}\left(\hat{a},\hat{b}\right)-E_{HV}\left(\hat{a},\hat{c}\right)\right|\leq 1+E_{HV}\left(\hat{b},\hat{c}\right)$, and remembering that the probabilities P must be positive, we obtain the following inequality:

$$\left|2P_1+2P_3\right|\leq 2P_1+2P_2. \tag{8.2}$$

This inequality is violated for all values of P_1 and P_4, and for $P_2 < P_3$. This is graphically represented by the light area in the graph below. Remarkably, this demonstrates that if $P_2 < P_3$, Bell's Inequality is violated and the system therefore has quantum attributes for *any* values of P_1 and P_4. These are the kinds of relationships we wish to investigate with our SIMPLEX computer model.

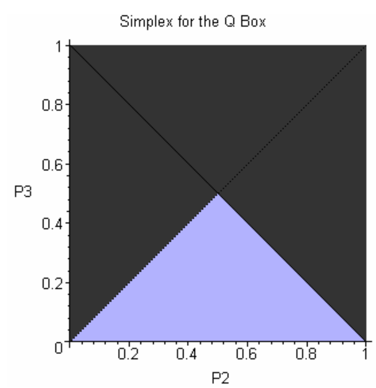


Figure 8-4 The simplex showing the region for which Bell's Theorem is violated.

8.3 The MATLAB Program SIMPLEX

SIMPLEX uses the same shell as the KEEPER program, but has been modified so that some configurations are more likely to occur in the loading of the special cubes. The probabilities are denoted by P_1, P_2, P_3 and P_4.

Cube 1	↑ →	↑ ←	↓ →	↓ ←
Cube 2	↓ ←	↓ →	↑ ←	↑ →
Probability	P_1	P_2	P_3	P_4

Figure 8-5 Possible configurations for the inside and outside cube and their associated probabilities P_n.

The SIMPLEX program is designed to test the different regions of our simplex in which it is predicted that Bell's Theorem is always violated. This is accomplished with a simple weighting algorithm during the portion of the program that controls the loading of the cube. A sample SIMPLEX run is included in the appendices.

8.4 Insights Gleaned

Programming KEEPER and SIMPLEX was a valuable exercise since it forced us to think about what it means to be 'entangled' and 'nonlocal', and how one would turn these concepts into computer code. During the process I asked several professors what it means to be 'entangled', and how one would program an entangled system. The best and only really feasible advice received was, “This is how you do entanglement. Don’t look!”[14]

And so we “didn’t look”. We set up two sets of cubes that are shuffled in tandem. Redcube and greencube are reference cubes that contain the hidden variables of 0.1 (spin-up), 0.2 (spin-down), 0.3 (spin-right), and 0.4 (spin-left). Entangled Redcube and Entangled Greencube are the result of rounding down the hidden variables so that they are all zero. Redcube and greencube are set aside at the beginning of the program and not accessed again until measurement time.

We can decide at any time to “peek” during the shuffling of the cube. If we decide to have the computer report the positions of all of the minicubes that make up Entangled Redcube or Entangled Greencube, the computer will report a 3x3x3 matrix (row, column and page) filled with mixed up numbers 2 through 26 (less one of them that is outside) and two zeros. If one of the ‘entangled’ minicubes is ‘outside’ the cube when we choose to look, then there is only one zero in the 3x3x3 matrix map produced by the computer, and the value of the outside cube is reported to be zero. Everything is programmed to look entangled, *but the Entangled Cubes are just a façade*! When a measurement is triggered by a zero popping out of Entangled Redcube, we then search for the other zero we know must be inside. These coordinates are used to access the same position on Redcube. Redcube, remember, is the repository of the hidden variables, and has been shuffled at the same time using the same random shuffling sequences. The measurements are made and the rules applied which should *force* the inequality. And they do.

[14]This advice was from Dr. Ross Spencer, accompanied by a pantomime of “See No Evil”.

The realization that the program worked very well to violate Bell's Inequality with or without the elaborate entanglement process was a key insight. We realized that it is not necessary to "hide" the hidden variables. The hidden variables can be assigned openly as strict correlations from the beginning. The key to turning our classical system into one which behaves in a quantum manner is that these hidden variables must be subject to change based on a nonlocal measurement.

Chapter 9

Comparisons

In this chapter we would like to discuss how our model of nonlocal hidden variables differs from other quantum theories. David Griffiths [28] does an excellent job of summing up the doctrinal viewpoints of the different theories from which quantum physicists can choose when discussing where the particle was just before measurement:

> 1. *The **realist** position: The particle was at C. This certainly seems like a sensible response, and it is the one Einstein advocated. Note, however, that if this is true then quantum mechanics is an incomplete theory, since the particle really was at C, and yet quantum mechanics was unable to tell us so. To the realist, indeterminacy is not a fact of nature, but a reflection of our ignorance. As d'Espagnat put it, "The position of the particle was never indeterminate, but was merely unknown to the experiment."* [33] *Evidently* Ψ *is not the whole story—some additional information (known as a hidden variable) is needed to provide a complete description of the particle.*
>
> 2. *The **orthodox** position: The particle wasn't really anywhere. It was the act of measurement that forced the particle to "take a stand" (though how and why it decided on the point C we dare not ask). Jordan said it most starkly: "Observations not only disturb what is to be measured, they produce it. . . We compel [the particle to assume a definite position."* [34] *This view (the so-called **Copenhagen interpretation**) is associated with Bohr and his followers. Among physicists it has always been the most widely accepted position. Note, however, that if it is correct there is something very peculiar about the act of measurement—something that over half a century of debate has done precious little to illuminate.*
>
> 3. *The agnostic position: Refuse to answer. This is not quite as silly as it sounds—after all, what sense can there be in making assertions about the status of a particle before a measurement, when the only way of knowing whether you were right is precisely to conduct a measurement, in which case what you get is no longer "before the measurement"? It is metaphysics (in the pejorative sense of the word) to*

worry about something that cannot, by its nature, be tested. Pauli said, "One should no more rack one's brain about the problem of whether something one cannot know anything about exists all the same, than about the ancient question of how many angels are able to sit on the point of a needle." For decades this was the "fall-back" position of most physicists: They'd try to sell you answer 2, but if you were persistent they'd switch to 3 and terminate the conversation.

To these three physics schools of thought we would like to add one more ideological framework to mold one's musings:

4. *The quantum theory proposed by David Bohm.* [35]

We wish to specifically address David Bohm's Quantum Theory, since it is also fundamentally nonlocal and has hidden variables. Throughout our research we have assumed nonlocality as embodied in our Q Box joint measurement and the accompanying rules which act to change the hidden variables such that the probability space morphs from classical to quantum. We have assigned hidden variables that are intrinsic characteristics carried by the "particles" in our model. (The 'particles' in the Q Box system are the two special mini-cubes.) Let's compare that with Bohmian Mechanics, as put forward by David Bohm. [35]

9.1 Bohmian Mechanics

In 1952 David Bohm [35] produced a remarkably elegant answer to the philosophically vexing Quantum Court of Copenhagen. He began conventionally, with the Schrödinger equation and the quantum wave function Ψ in polar coordinates:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \tag{9.1}$$

$$\psi = \text{R}e^{\left(iS/\hbar\right)} \tag{9.2}$$

By inserting Ψ into the Schrödinger equation and separating out the real and imaginary parts he comes up with two separate equations:

$$\frac{\partial R^2}{\partial t} + \nabla \cdot \left(R^2 \frac{\nabla S}{m} \right) = 0 \tag{9.3}$$

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} = 0. \tag{9.4}$$

The second (imaginary) part looks like a classical Hamilton-Jacobi equation for a particle with momentum $\vec{p} = \nabla S$ except that it started out imaginary and has a small ∇^2 term. In the WKB approximation of standard quantum mechanics (where the wave packet width is considered to be much greater than the wave length), this small term would be considered too small to contribute significantly and would be thrown out. Bohm keeps it instead, calls it the quantum potential *Q*, and rewrites the equation with a radical new two-part potential:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0 \tag{9.5}$$

So, by beginning at the beginning, making a mathematical manipulation and deciding the dross was really gold, Bohm was able to *reinterpret* quantum mechanics. In his formalism, the particle acts classically. (Almost! We will come back to this!) It has a trajectory and a well-defined position in space. Bohmian particles are guided by the quantum potential *Q*, like little surfers riding a wave. Indeed, Bohm calls his quantum potential “the pilot wave.”

Previous to Bohm and his quantum potential, it seemed that the only possible explanation for the results of the double slit experiment was that particles not only interfered with each other but with themselves. This gave the Copenhagen Camp experimental substantiation for their position that the particle was both places at once; thus the quantum orthodox tenet of indeterminacy. Using his pilot wave potential, though, Bohm was able to theoretically predict and duplicate the results of the double slit experiment without resorting to indeterminacy.

Bohmian Quantum Mechanics can be thought of as an affirmative answer to Einstein’s question of whether quantum mechanics is incomplete. “Yes”, Bohm seems to say. “The theory *is* incomplete. See, you just missed this part! If you massage the equation here and keep this there, look what pops out!” On the other hand, since Bohm started “from the beginning” as it were, and did nothing more than manipulate and reinterpret what was already there, it can also be argued that Standard Quantum is complete but the practitioners are blind to all of its possibilities.

There is a catch. As we mentioned previously, the particle in the Bohmian interpretation acts classically, with a well-defined position and trajectory. But the particle has a very nonclassical attribute: It is profoundly nonlocal. The quantum potential that propels the particle along is a function of the position *of every other particle in the system,* and the nonlocality goes even deeper than that. Since Ψ is the definitive description of a Bohmian quantum system just as it is in a standard quantum system, the quantum potential must also be a function of every possible atomic state of the particle(s). Every possible state and position of each particle is therefore entangled with every possible state and position of every other particle in the system. By extension, every system is entangled with every other system. This leads to a world-view that is so completely intertwined that it can only be considered as a whole, hence the title of Bohm's book, *The Undivided Universe.* [35] While Bell's Theorem has made many scientists feel more comfortable with the idea of nonlocality, Bohmian Quantum Mechanics takes nonlocality to a whole new level.[15]

9.2 Comparison to Bohm

Our model differs from Bohmian Mechanics from its very inception. Bohm started with wave functions and the Schrödinger equation. We begin with expectation values from a probability theory perspective and then use Bell's Theorem as a test for quantum attributes in our system. Nowhere in our model is there a Ψ to be seen.

Like Bohmian Mechanics, we find nonlocality to be an integral part of quantum systems. Our Q Box rules, which are applied upon measurement, are the equivalent of Bohm's pilot wave. Our rules tell our "particles" (the special cubes), which are classical in every way excepting their nonlocal entanglement, how they must act if they are to behave in a nonclassical way. The particles then prove their quantumness by violating Bell's Inequality. Bohm's pilot wave serves the same purpose in his theory. The quantum potential, or pilot wave, tells his particles, which are also classical in every way excepting their nonlocal entanglement, how they must behave if they are to be quantum and form the expected double slit pattern.

[15] Even Bell calls Bohmian Mechanics 'a grossly non-local structure'. [21]

The mechanism by which a Bohmian system transforms from quantum to classical differs from the process that governs our model's transformation in the classical limit. In a Bohmian system, the nonlocality is contained within the quantum potential and is too small to be detected in a macroscopic system. Bohm says [21]:

> *. . . Why [is it that] nonlocality is not encountered in our common experience of the world? Basically the answer to this question is quite simple. For our ordinary experience, both in the domain of common sense and in that of classical physics, is restricted to situations in which the quantum potential is very small, so that, in this context at least, it does not produce significant EPR correlations. For, as we have seen, quantum nonlocality is entirely the product of the quantum potential.*

On the other hand, in our model nonlocality is assumed to be pervasive in the system. We assume when making our $\hat{c}$ measurement that it is possible to make an instantaneous joint measurement between the two special cubes. This type of measurement is expressly nonlocal.

In other words, in our model nonlocality is embodied in the combination of the strict correlation (entanglement) of our system and the application of the rules that change the hidden variables of our system. We believe that macroscopic systems do not manifest quantum entanglement because the effect has been washed out through marginalization. This is reminiscent of the information lost upon calculation of $|\Psi(x,t)|^2$ in a standard quantum problem. The process of taking the absolute value squared obliterates the phase information contained in Ψ.

9.3 Comparison to the Copenhagen Interpretation

The Copenhagen Interpretation of quantum mechanics begins with the wave function Ψ, which represents all of the information that is possible to know about the particle. In a spin-½ system, for example, the particle is considered to be both spin-up and spin-down until measurement, when it collapses into one or the other. Premeasurement, the particle can be thought of as having the *potential* to be either spin-up or spin-down.

Our Q Box hidden variables can be thought of as also being indeterminate in a way, even though we begin with the hidden variables hardcoded into the two special cubes. Let's reexamine Figs. 5.7 and 5.8 from Chap. 5. These two tables enumerated the results of our measurements before and after the application of our Q Box Rules:

1) If $\hat{a}$ *and* $\hat{b}$ are being measured jointly together, their values remain unchanged.

2) If $\hat{a}$ is being measured with $\hat{c}$, its spin value is forced to match that of $\hat{c}$.

3) If $\hat{b}$ is being measured with $\hat{c}$, its spin value is forced to be opposite that of $\hat{c}$.

The bottom table in Figure 9-1 shows again how $\hat{a}$ and $\hat{b}$ either remain in their classical form or change to show their quantum nature as required by the nonlocal $\hat{c}$ measurement. Because there is for every realization of $\hat{a}$ and $\hat{b}$ a possibility that it will have to change to meet the conditions specified in the rules, $\hat{a}$ and $\hat{b}$ can be thought of as being classically +1 or -1, but with the quantum *potential* of being the opposite value; they begin life as -1 or +1, but it is possible that they may have to change. The important point here is that when looking at only the end result, there is no way to tell our Q Box particles from those expected in any other quantum system.

Cube #1 Cube #2	↑ → ↓ ←	↑ ← ↓ →	↓ → ↑ ←	↓ ← ↑ →
$\hat{a}$	+1	+1	-1	-1
$\hat{b}$	-1	+1	-1	+1
$\hat{c}$	-1	+1	+1	-1
$\hat{a} * \hat{b}$	-1	+1	+1	-1
$\hat{b} * \hat{c}$	+1	+1	-1	-1
$\hat{a} * \hat{c}$	-1	+1	-1	+1

<table>
<tr><td>Cube #1
Cube #2</td><td colspan="2">↑ →
↓ ←</td><td colspan="2">↑ ←
↓ →</td><td colspan="2">↓ →
↑ ←</td><td colspan="2">↓ ←
↑ →</td></tr>
<tr><td rowspan="2">$\hat{a}$</td><td rowspan="2">+1</td><td>With b
+1</td><td rowspan="2">+1</td><td>With b
+1</td><td rowspan="2">-1</td><td>With b
-1</td><td rowspan="2">-1</td><td>With b
-1</td></tr>
<tr><td>With c
-1</td><td>With c
+1</td><td>With c
+1</td><td>With c
-1</td></tr>
<tr><td rowspan="2">$\hat{b}$</td><td rowspan="2">-1</td><td>With a
-1</td><td rowspan="2">+1</td><td>With a
+1</td><td rowspan="2">-1</td><td>With a
-1</td><td rowspan="2">+1</td><td>With a
+1</td></tr>
<tr><td>With c
+1</td><td>With c
-1</td><td>With c
-1</td><td>With c
+1</td></tr>
<tr><td>$\hat{c}$</td><td colspan="2">-1</td><td colspan="2">+1</td><td colspan="2">+1</td><td colspan="2">-1</td></tr>
<tr><td>$\hat{a} * \hat{b}$</td><td colspan="2">-1</td><td colspan="2">+1</td><td colspan="2">+1</td><td colspan="2">-1</td></tr>
<tr><td>$\hat{b} * \hat{c}$</td><td colspan="2">-1</td><td colspan="2">-1</td><td colspan="2">-1</td><td colspan="2">-1</td></tr>
<tr><td>$\hat{a} * \hat{c}$</td><td colspan="2">+1</td><td colspan="2">+1</td><td colspan="2">+1</td><td colspan="2">+1</td></tr>
</table>

Figure 9-1 Measurement values *before and after* application of the Q Box Rules.

We wish to stress that our model in no way refutes the Orthodox View.[16] Our Q Box Model still produces quantum results; but in an alternative way from the Standard Model. In our next, concluding chapter we will discuss the insights gleaned from using this unconventional approach.

[16] I feel like a kindred spirit to John Bell, who said, "We emphasize not only that our view is that of a minority, but also that current interest in such questions is small. The typical physicist feels that they have long been answered, and that he will fully understand just how if ever he can spare twenty minutes to think about it." [21]

Chapter 10

Conclusions

N. David Mermin, my favorite writer on all things physics, has had much to say about nonlocality over the years. He mostly doesn't like it— he considers nonlocality to be "too cheap a way out of the deep [quantum] conundra."[36] Mermin suggests that instead of nonlocality, the physics community would be better served to call the phenomenon the SF mechanism, harking back to 'spukhafte Fernwirkungen', Einstein's original term for spooky action at a distance. Mermin argues that in switching to a more evocative name scientists will be more apt to see the quantum curiosity of nonlocality as "a 'trick'. . . reminiscent of stage magic, which works by a process of self-deception, skillfully helped along by the magician. When the trick is SF, the magician is Nature herself." [36]

Given that this whole thesis is based on showing that quantum effects can be had with the right combination of hidden variables and a nonlocal twist, one would think that we would have nothing in common with N. David Mermin. But I find that a statement by Mermin made in the same article that included the jibes quoted above sums my conclusions about physical reality, based on this research, better than any other I have seen or heard or read. Mermin said:

> *I believe that the reconciliation [of the quantum conundra] is to be found in a view of physical reality as comprised only of correlations*

between different parts of the physical world, and not at all of unconditional properties possessed by those parts. In that case the straight answer to the question posed in my title, [What Do These Correlations Know about Reality?] is . . . "everything." And the straight question—to which the straight answer is indeed "nothing"—is, "What do these correlations know about individual properties?" [36]

Having said this, it is time to discuss what our research really shows.

10.1 Results of Research

This research was built around Bell's Inequality. Bell's Theorem proves that any given *local* system that includes *hidden variables* must abide by the inequality. We have made use of the flip side of Bell's Theorem: any system that includes *hidden variables* and violates the inequality must therefore be *nonlocal.* And since quantum systems are the only such systems found thus far in nature, we have used the violation of Bell's Theorem as an acid test for quantum qualities.

With Bell's Theorem we have shown how one might take a classical system, our Q Box, and make it behave in a quantum manner. This transformation requires a nonlocal connection that in some way causes a change in the manifestation of the hidden variables. We have no way of knowing the mechanism of this change. Although we indulged in some entertaining speculation in Chap. 5 with our Quantum Maxwellian Demons and Spaceports, this was primarily pedagogical; an effort to find some illumination to banish the 'spookiness'.

We used the same technique on Mermin's Box, which is the thought experiment Mermin designed to illustrate the 'quantum conundrum', the impossibility of having hidden variables in a quantum object. We were able to show that with a suitable combination of hidden variables and nonlocal connections we can cure Mermin's 'quantum conundrum' and provide just such an 'impossible' construct.

10.2 Caveats

Our model speaks only about systems with hidden variables. Our model in no way invalidates the standard quantum status quo. It is consistent with both the Standard Interpretation and Bohmian Mechanics in terms of the final results. Our research explores other ways of looking at the mysteries of quantum systems.

10.3 Last words

Throughout this research there has been one salient fact, one shining feature of nature that has become blindingly clear to us. It is as Mermin stated: The *correlation* of a system is the actual Reality; not the individual properties, not the hidden variables. It is the connectedness that is the more fundamental truth. We feel the same as Bell when he said, "Perhaps Nature is not so queer as quantum mechanics. But the experimental situation is not very encouraging from this point of view." [21]

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Appendix

<u>**KEEPER Computer Program**</u>:

```
2/9/06 12:12 PM U:\thesis\KEEPER.m 1 of 5
close
%I have figured out how to violate Bell's Theorem |E(a,b)-
E(a,c)|<=1+E(b,c),with a series
of
%measurements made on my cube. Ahat is the vertical measurement on cube
%#1, call it V1. Bhat is the horizontal measurement on cube #2, call it H2.
Chat is the
%combination by LHRule of the horizontal on cube #1 and the vertical on
%cube #2, or H1xV2. Then I need the E1(a,b), E1(a,c) and E1(b,c). This is
the + or -
%value of the product of the two measurements. There is a Rule then
%applied. If measurement a and measurement b are both posittive (yielding
%an overall positive value, then the value of ahatxbhat and the value of
%ahatxchat flips. If the values of both ahat and bhat are negagtive, then
%both ahatxbhat and bhatxchat flip signs. So, I need to be able to find:
%V1, V2, H1, H2, LHval, cval=H1[xbyLHRule]V2, ab=V1xH2, bc=H2xcval,
%ac=V1xcval
x0red y0red z0red ared bred cred
x0green y0green z0green agreen bgreen cgreen
count Vin Vout Hin Hout abtot bctot actot cval
Niterations H1 H2 V1 V2
count
s
for s=1:1
m
for z=1:3 %Loading the cube 1-27
for y=1:3
for x=1:3
m=m
redcube(x,y,z)=m
greencube(x,y,z)=m
end
end
end
%The red matrix tracks the spin up and down "red" characteristic of the
%cubes.
redqerase=floor(2*rand) %Smearing out the loading of spin up and spin down
%This code loads a spin up and spin down into
%the cube into the two corners, but I don't know which
%is in which place
if redqerase==0
redcube(1,1,1) %This will be 'spin up'
redcube(3,3,3) %This will be 'spin down'
elseif redqerase==1
redcube(1,1,1)
redcube(3,3,3)
end
redcube
%The green matrix tracks the spin left and right "green" characteristic of
%the cubes.
```

```
greenqerase=floor(2*rand) %Smearing out the loading of spin up and spin
down
%This code makes spin up and spin down loaded into
%the cube into the two corners, but I don't know which
2/9/06 12:12 PM U:\thesis\KEEPER.m 2 of 5
%is in which place
if greenqerase==0
greencube(1,1,1) %This will be 'spin right'
greencube (3,3,3) %This will be 'spin left'
elseif greenqerase==1
greencube(1,1,1)
greencube(3,3,3)
end
Greencube
Entangledgreencube=floor(greencube) %This code makes both the spin up and
%spin down the value of zero in the matrix. They are
%indistinguishable in this matrix. I know they
%start in the corners, but I don't know which one is
%in which corner
entangledredcube=floor(redcube)
redin %The purple ball is 'zero '
greenin
xhat='xhat'
negxhat='-xhat'
yhat='yhat'
negyhat='-yhat'
zhat='zhat'
negzhat='-zhat'
for n=1:200 %Number of iterations
mod(n)=floor(2*rand)
face(n)=ceil(6*rand) %Choosing a face to push on
x(n)=ceil(3*rand) % And a position on that face
y(n)=ceil(3*rand)
z(n)=ceil(3*rand)
if x(n)==2&y(n)==2&z(n)==2 %[2,2,2] not allowed
x(n)=ceil(3*rand)
y(n)=ceil(3*rand)
z(n)=ceil(3*rand)
else
iteration=n %Tracking iterations
pushpoint=[x(n),y(n),z(n)] %Printing pushpoint if I need to
%The shuffling mechanics is in the following section:
if face(n)==1 %-zhat
redout=redcube(x(n),y(n),1)
redcube(x(n),y(n),1)=redcube(x(n),y(n),2)
redcube(x(n),y(n),2)=redcube(x(n),y(n),3)
redcube(x(n),y(n),3)=redin
greenout=greencube(x(n),y(n),1)
greencube(x(n),y(n),1)=greencube(x(n),y(n),2)
greencube(x(n),y(n),2)=greencube(x(n),y(n),3)
greencube(x(n),y(n),3)=greenin
% disp(negzhat)
elseif face(n)==2 %-xhat
redout=redcube(1,y(n),z(n))
2/9/06 12:12 PM U:\thesis\KEEPER.m 3 of 5
redcube(1,y(n),z(n))=redcube(2,y(n),z(n))
redcube(2,y(n),z(n))=redcube(3,y(n),z(n))
redcube(3,y(n),z(n))=redin
greenout=greencube(1,y(n),z(n))
greencube(1,y(n),z(n))=greencube(2,y(n),z(n))
greencube(2,y(n),z(n))=greencube(3,y(n),z(n))
greencube(3,y(n),z(n))=greenin
% disp(negxhat)
elseif face(n)==3 %-yhat
redout=redcube(x(n),1,z(n))
redcube(x(n),1,z(n))=redcube(x(n),2,z(n))
redcube(x(n),2,z(n))=redcube(x(n),3,z(n))
redcube(x(n),3,z(n))=redin
```

```
greenout=greencube(x(n),1,z(n))
greencube(x(n),1,z(n))=greencube(x(n),2,z(n))
greencube(x(n),2,z(n))=greencube(x(n),3,z(n))
greencube(x(n),3,z(n))=greenin
% disp(negyhat)
elseif face(n)==4 %zhat
redout=redcube(x(n),y(n),3)
redcube(x(n),y(n),3)=redcube(x(n),y(n),2)
redcube(x(n),y(n),2)=redcube(x(n),y(n),1)
redcube(x(n),y(n),1)=redin
greenout=greencube(x(n),y(n),3)
greencube(x(n),y(n),3)=greencube(x(n),y(n),2)
greencube(x(n),y(n),2)=greencube(x(n),y(n),1)
greencube(x(n),y(n),1)=greenin
% disp(zhat)
elseif face(n)==5 %xhat
redout=redcube(3,y(n),z(n))
redcube(3,y(n),z(n))=redcube(2,y(n),z(n))
redcube(2,y(n),z(n))=redcube(1,y(n),z(n))
redcube(1,y(n),z(n))=redin
greenout=greencube(3,y(n),z(n))
greencube(3,y(n),z(n))=greencube(2,y(n),z(n))
greencube(2,y(n),z(n))=greencube(1,y(n),z(n))
greencube(1,y(n),z(n))=greenin
% disp(xhat)
elseif face(n)==6 %yhat
redout=redcube(x(n),3,z(n))
redcube(x(n),3,z(n))=redcube(x(n),2,z(n))
redcube(x(n),2,z(n))=redcube(x(n),1,z(n))
redcube(x(n),1,z(n))=redin
greenout=greencube(x(n),3,z(n))
greencube(x(n),3,z(n))=greencube(x(n),2,z(n))
greencube(x(n),2,z(n))=greencube(x(n),1,z(n))
greencube(x(n),1,z(n))=greenin
% disp(yhat)
end
end
if redout<1
Vout=redout
```

```
Hout=greenout
count=count
if Vout==.1
V1
Verticalmeasurement='Spin up' %outside is vertical
elseif Vout==.2
V1
Verticalmeasurement='Spin Down'
end
if Hout==.3
H1
Horizontalmeasurement='Spin right'
elseif Hout==.4
H1
Horizontalmeasurement='Spin Left'
end
end
for x=1:3
for y=1:3
for z=1:3
if redcube(x,y,z)<1
x0red=x y0red=y z0red=z
end
end
end
end
endposition=[x0red,y0red,z0red]
Vin=redcube(x0red,y0red,z0red)
```

```
%Assigning values to the ups and downs, rights and lefts.
if Vin==.1
V2
elseif Vin==.2
V2
end
Hin=greencube(x0red,y0red,z0red)
if Hin==.3
H2
elseif Hin==.4
H2
end
%Applying the rule: If measurement a (V1) and measurement b (H2) are both
posittive
(yielding
%an overall positive value, then the value of ahatxbhat (cval) and the
value of
%ahatxchat flips. If the values of both ahat and bhat are negagtive, then
%both ahatxbhat and bhatxchat flip signs.
if H1==1 & V2==-1
cval
```



```
end
if H1==-1 & V2==1
cval
end
if H1==-1 & V2==-1
cval
end
if H1==1 & V2==1
cval
end
ab=V1*H2
bc=H2*cval
ac=V1*cval
newab=ab
abtot=abtot+newab
newbc=bc
bctot=bctot+newbc
newac=ac
actot=actot+newac
redin=redout
greenin=greenout %Tracking the ball outside
s=s
end
end
count
abtot
bctot
actot
Eab=abtot/count
Ebc=bctot/count
Eac=actot/count
BellsRHS=1+Ebc
BellsLHS=abs(Eab-Eac)
if BellsLHS>BellsRHS
disp('Bell''s Inequality Violated')
else disp('Bells''s Inequality Satisfied!')
end
```

KEEPER Results:

```
2/9/06 1:22 PM MATLAB Command Window 1 of 15
count =
157
abtot =
-166
bctot =
74
actot =
348
Eab =
-1.0573
Ebc =
0.4713
Eac =
2.2166
Bell's Inequality Violated
count =
164
abtot =
93
bctot =
-185
2/9/06 1:22 PM MATLAB Command Window 2 of 15
actot =
115
Eab =
0.5671
Ebc =
-1.1280
Eac =
0.7012
Bell's Inequality Violated
count =
138
abtot =
316
bctot =
246
actot =
140
Eab =
2.2899
Ebc =
1.7826
Eac =
2/9/06 1:22 PM MATLAB Command Window 3 of 15
1.0145
Bells's Inequality Satisfied!
count =
149
abtot =
222
bctot =
472
actot =
108
Eab =
1.4899
Ebc =
3.1678
Eac =
0.7248
Bells's Inequality Satisfied!
count =
181
```

```
abtot =
-147
bctot =
```

```
-123
actot =
161
Eab =
-0.8122
Ebc =
-0.6796
Eac =
0.8895
Bell's Inequality Violated
count =
139
abtot =
283
bctot =
283
actot =
409
Eab =
2.0360
Ebc =
```

```
2.0360
Eac =
2.9424
Bells's Inequality Satisfied!
count =
152
abtot =
-257
bctot =
115
actot =
-161
Eab =
-1.6908
Ebc =
0.7566
Eac =
-1.0592
Bells's Inequality Satisfied!
count =
145
abtot =
```

```
-124
bctot =
-322
actot =
446
Eab =
-0.8552
Ebc =
-2.2207
Eac =
3.0759
Bell's Inequality Violated
count =
181
abtot =
-185
bctot =
21
actot =
```

```
-167
Eab =
-1.0221
```

```
Ebc =
0.1160
Eac =
-0.9227
Bells's Inequality Satisfied!
count =
160
abtot =
43
bctot =
-47
actot =
353
Eab =
0.2687
Ebc =
-0.2938
Eac =
2.2062
Bell's Inequality Violated
count =
167
```

```
abtot =
-138
bctot =
-94
actot =
120
Eab =
-0.8263
Ebc =
-0.5629
Eac =
0.7186
Bell's Inequality Violated
count =
170
abtot =
-47
bctot =
-179
actot =
103
```

```
Eab =
-0.2765
Ebc =
-1.0529
Eac =
0.6059
Bell's Inequality Violated
count =
145
abtot =
-5
bctot =
-379
actot =
-243
Eab =
-0.0345
Ebc =
-2.6138
```

Eac =
-1.6759
Bell's Inequality Violated

count =
133
abtot =
76
bctot =
-412
actot =
-400
Eab =
0.5714
Ebc =
-3.0977
Eac =
-3.0075
Bell's Inequality Violated
count =
144
abtot =
-65
bctot =
-69

actot =
-133
Eab =
-0.4514
Ebc =
-0.4792
Eac =
-0.9236
Bells's Inequality Satisfied!
count =
146
abtot =
139
bctot =
513
actot =
-79
Eab =
0.9521
Ebc =
3.5137
Eac =

-0.5411
Bells's Inequality Satisfied!
count =
150
abtot =
80
bctot =
148
actot =
-84
Eab =
0.5333
Ebc =
0.9867
Eac =
-0.5600
Bells's Inequality Satisfied!
count =
155

```
abtot =
-412
bctot =
2/9/06 1:22 PM MATLAB Command Window 13 of 15
-182
actot =
258
Eab =
-2.6581
Ebc =
-1.1742
Eac =
1.6645
Bell's Inequality Violated
count =
145
abtot =
-220
bctot =
204
actot =
-246
Eab =
-1.5172
Ebc =
2/9/06 1:22 PM MATLAB Command Window 14 of 15
1.4069
Eac =
-1.6966
Bells's Inequality Satisfied!
count =
182
abtot =
10
bctot =
84
actot =
-168
Eab =
0.0549
Ebc =
0.4615
Eac =
-0.9231
Bells's Inequality Satisfied!
count =
161
abtot =
2/9/06 1:22 PM MATLAB Command Window 15 of 15
150
bctot =
-56
actot =
-296
Eab =
0.9317
Ebc =
-0.3478
Eac =
-1.8385
Bell's Inequality Violated
>>
```

SIMPLEX Computer Program:

```
close
%To get this weighted version of the program I will catch the cube every
%time it comes out. As it is now, each configuration, #1, #2, #3, and #4
%are equally likely. So I will have count1, count2, count3, and count4.
%When count1=1 (everytime config #1 is encountered) I will measure it.
%Configuration #2 will have to be counted twice (count2=2) before it is
%measured. Measurement will happen for count3=4 and count4=4. This should
%provide the optimum violation of Bell's.
x0red y0red z0red ared bred cred
x0green y0green z0green agreen bgreen cgreen
count Vin Vout Hin Hout abtot bctot actot cval
Niterations H1 H2 V1 V2 count1 count2 count3 count4
count ab bc ac p1 p2 p3 p4 weight purple
s
for s=1:100
m
for z=1:3 %Loading the cube 1-27
for y=1:3
for x=1:3
m=m
redcube(x,y,z)=m
greencube(x,y,z)=m
end
end
end
%The red matrix tracks the spin up and down "red" characteristic of the
%cubes.
redqerase=floor(2*rand) %Smearing out the loading of spin up and spin down
%This code loads a spin up and spin down into
%the cube into the two corners, but I don't know which
%is in which place
if redqerase==0
redcube(1,1,1) %This will be 'spin up'
redcube(3,3,3) %This will be 'spin down'
elseif redqerase==1
redcube(1,1,1)
redcube(3,3,3)
end
redcube
%The green matrix tracks the spin left and right "green" characteristic of
%the cubes.
greenqerase=floor(2*rand) %Smearing out the loading of spin up and spin
down
%This code makes spin up and spin down loaded into
%the cube into the two corners, but I don't know which
%is in which place
if greenqerase==0
greencube(1,1,1) %This will be 'spin right'
greencube(3,3,3) %This will be 'spin left'
elseif greenqerase==1
greencube(1,1,1)
```



```
greencube(3,3,3)
end
greencube
entangledgreencube=floor(greencube) %This code makes both the spin up and
%spin down the value of zero in the matrix. They are
%indistinguishable in this matrix. I know they
%start in the corners, but I don't know which one is
%in which corner
entangledredcube=floor(redcube)
redin %The purple ball is 'zero '
greenin
for n=1:200 %Number of iterations
mod(n)=floor(2*rand)
```

```
face(n)=ceil(6*rand) %Choosing a face to push on
x(n)=ceil(3*rand) % And a position on that face
y(n)=ceil(3*rand)
z(n)=ceil(3*rand)
if x(n)==2&y(n)==2&z(n)==2 %[2,2,2] not allowed
x(n)=ceil(3*rand)
y(n)=ceil(3*rand)
z(n)=ceil(3*rand)
else
iteration=n %Tracking iterations
pushpoint=[x(n),y(n),z(n)] %Printing pushpoint if I need to
%The shuffling mechanics is in the following section:
if face(n)==1 %-zhat
redout=redcube(x(n),y(n),1)
redcube(x(n),y(n),1)=redcube(x(n),y(n),2)
redcube(x(n),y(n),2)=redcube(x(n),y(n),3)
redcube(x(n),y(n),3)=redin
greenout=greencube(x(n),y(n),1)
greencube(x(n),y(n),1)=greencube(x(n),y(n),2)
greencube(x(n),y(n),2)=greencube(x(n),y(n),3)
greencube(x(n),y(n),3)=greenin
% disp(negzhat)
elseif face(n)==2 %-xhat
redout=redcube(1,y(n),z(n))
redcube(1,y(n),z(n))=redcube(2,y(n),z(n))
redcube(2,y(n),z(n))=redcube(3,y(n),z(n))
redcube(3,y(n),z(n))=redin
greenout=greencube(1,y(n),z(n))
greencube(1,y(n),z(n))=greencube(2,y(n),z(n))
greencube(2,y(n),z(n))=greencube(3,y(n),z(n))
greencube(3,y(n),z(n))=greenin
% disp(negxhat)
elseif face(n)==3 %-yhat
redout=redcube(x(n),1,z(n))
redcube(x(n),1,z(n))=redcube(x(n),2,z(n))
redcube(x(n),2,z(n))=redcube(x(n),3,z(n))
```



```
redcube(x(n),3,z(n))=redin
greenout=greencube(x(n),1,z(n))
greencube(x(n),1,z(n))=greencube(x(n),2,z(n))
greencube(x(n),2,z(n))=greencube(x(n),3,z(n))
greencube(x(n),3,z(n))=greenin
% disp(negyhat)
elseif face(n)==4 %zhat
redout=redcube(x(n),y(n),3)
redcube(x(n),y(n),3)=redcube(x(n),y(n),2)
redcube(x(n),y(n),2)=redcube(x(n),y(n),1)
redcube(x(n),y(n),1)=redin
greenout=greencube(x(n),y(n),3)
greencube(x(n),y(n),3)=greencube(x(n),y(n),2)
greencube(x(n),y(n),2)=greencube(x(n),y(n),1)
greencube(x(n),y(n),1)=greenin
% disp(zhat)
elseif face(n)==5 %xhat
redout=redcube(3,y(n),z(n))
redcube(3,y(n),z(n))=redcube(2,y(n),z(n))
redcube(2,y(n),z(n))=redcube(1,y(n),z(n))
redcube(1,y(n),z(n))=redin
greenout=greencube(3,y(n),z(n))
greencube(3,y(n),z(n))=greencube(2,y(n),z(n))
greencube(2,y(n),z(n))=greencube(1,y(n),z(n))
greencube(1,y(n),z(n))=greenin
% disp(xhat)
elseif face(n)==6 %yhat
redout=redcube(x(n),3,z(n))
redcube(x(n),3,z(n))=redcube(x(n),2,z(n))
redcube(x(n),2,z(n))=redcube(x(n),1,z(n))
redcube(x(n),1,z(n))=redin
```

```
greenout=greencube(x(n),3,z(n))
greencube(x(n),3,z(n))=greencube(x(n),2,z(n))
greencube(x(n),2,z(n))=greencube(x(n),1,z(n))
greencube(x(n),1,z(n))=greenin
% disp(yhat)
end
end
if redout<1
Vout=redout
Hout=greenout
weight=rand
if Vout==.1 & Hout==.3 & weight<=.5
V1
V2
H1
H2
ab ac bc purple='p1'
```


```
% count=count+1
elseif Vout==.1 & Hout==.4 & .5<weight<=.75
V1
V2 purple='p2'
% Verticalmeasurement='Spin up'%outside is vertical
H1
H2
ab ac bc
% count=count+1
elseif Vout==.2 & Hout==.3 & .75<weight<.875
V1
V2
%Verticalmeasurement='Spin Down'
H1
H2
ab ac bc purple='p3'
count=count
elseif Vout==.2 & Hout==.4 & .875<weight<1
V1
V2
H1
H1
ab ac bc purple='p4'
count=count
else
V1 V2 H1 H2 purple='no measurement'
count=count
end
end
count
weight
purple
redin=redout
greenin=greenout %Tracking the ball outside
%count=count+1
s=s
newab=ab
abtot=abtot+newab
newbc=bc
bctot=bctot+newbc
newac=ac
actot=actot+newac
end
```


```
end
count
abtot
bctot
actot
Eab=abtot/count
```

```
Ebc=bctot/count
Eac=actot/count
BellsRHS=1+Ebc
BellsLHS=abs(Eab-Eac)
if BellsLHS>BellsRHS
disp('Bell''s Inequality Violated')
else disp('Bells''s Inequality Satisfied!')
end
```

Results for Simplex Probability Set #1. P1=.50, P2=.25, P3=.125, P4=.125

```
2/9/06 12:25 PM MATLAB Command Window 1 of 1
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bell's Inequality Violated
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
>>
```

Results for Simplex Probability Set #2P1=.25, P2=.25, P3=.25, P4=.25

```
2/9/06 12:27 PM MATLAB Command Window 1 of 1
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
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Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
Bells's Inequality Satisfied!
```

Results for Simplex Probability Set #3. P1=.95, P2=.01, P3=.01, P4=.03

```
2/9/06 12:39 PM MATLAB Command Window 1 of 1
Bell’s Inequality Satisfied!
Bell's Inequality Violated
Bell's Inequality Violated
Bell's Inequality Violated
Bell’s Inequality Satisfied!
Bell's Inequality Violated
Bell's Inequality Violated
Bell's Inequality Violated
Bell’s Inequality Satisfied!
Bell’s Inequality Satisfied!
Bell's Inequality Violated
Bell's Inequality Violated
Bell's Inequality Violated
Bell’s Inequality Satisfied!
Bell's Inequality Violated
Bell’s Inequality Satisfied!
Bell's Inequality Violated
Bell's Inequality Violated
Bell’s Inequality Satisfied!
Bell's Inequality Violated
Bell's Inequality Violated
Bell's Inequality Violated
Bell's Inequality Violated
Bell’s Inequality Satisfied!
Bell’s Inequality Satisfied!
Bell’s Inequality Satisfied!
Bell's Inequality Violated
Bell's Inequality Violated
Bell's Inequality Violated
Bell’s Inequality Satisfied!
>>
```